seen similar

A1 $\Pi = 2C_{20} - 3C_{30} + C_{50}$ ✓

$\Pi(t=T) = 2\max(S-20, 0)$

$-3\max(S-30, 0)$

When $t=T$ $+\max(S-50, 0)$ 3

If $0 < S < 20$

$\Pi = 0$

$P_{50} = 50 - S > \Pi$ 3

If $20 < S < 30$ $\quad S=20 \quad S=30$

$\Pi = 2S - 40 \quad (0 < \Pi < 20)$

$P_{50} = 50 - S \quad (20 < \Pi < 30)$

$S=30 \quad S=20$

$\Rightarrow P_{50} \geq \Pi$ 3

If $30 < S < 50$

$P_{50} = 50 - S, \quad (20 < P_{50} < 0$

$\Pi = -S + 50, \quad (20 > \Pi > 0$

$\Rightarrow P_{50} = \Pi$

3

If $S > 50$

$P_{50} = 0$

$\pi = 0$

$\Rightarrow P_{50} = \pi$

3

15

A ~~(i)~~ (i) no arbitrage $F = Se^{rT}$ (seen in class) ✓

$= 10 . e^{+0.75 \times .05}$

$= \$10.382$ 5

(ii) After 6 month

$25 \cancel{8} e^{0.5 \times .025} = 25.031$ 5

Then

$25.031 . e^{0.5 \times .04} = 25.5366$

– extension of (i) – no arbitrage

(iii) $F = 31500 . e^{0.5 \times (0.1 + 0.04)}$

$= 33784$

storage not seen before

– unseen (no arbitrage)

5

15

See Smith

A3 (i) $\pi = 2S - P(S, X)$

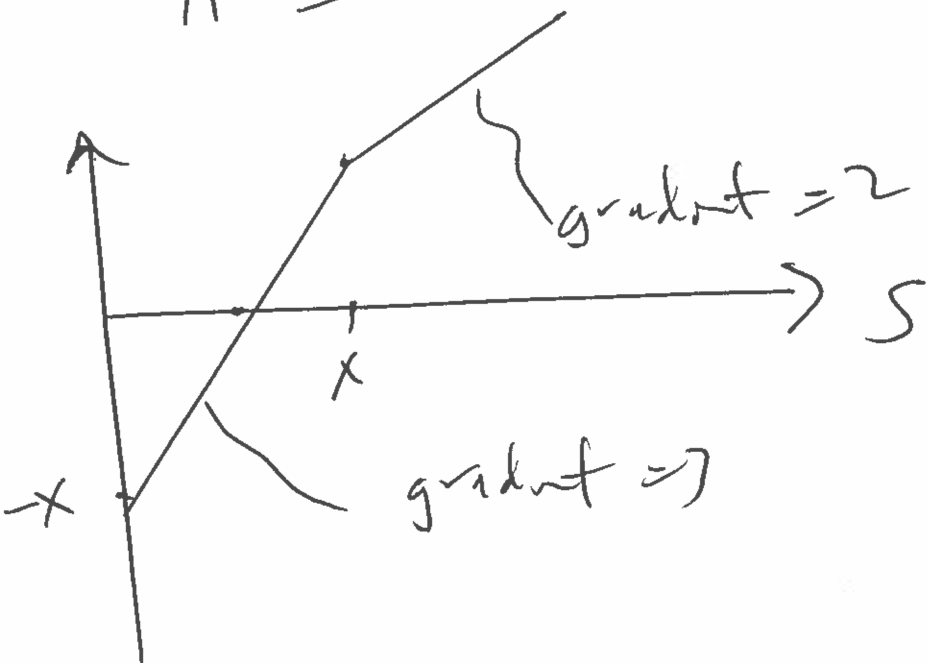
$\pi(t=T) = 2S - \max(X-S, 0)$

If $0 < S < X$

$\pi = 2S - X + S = 3S - X$

If $S > X$

$\pi = 2S$



(ii) $\pi = S - 2p(X_1) + c(X_2)$ 2

$\pi(t=T) = S - 2\max(X_1 - S, 0) + \max(S - X_2, 0)$

If $X_1 > X_2$

If $0 < S < X_2$

$\pi = S - 2(X_1 - S) = 3S - 2X_1$

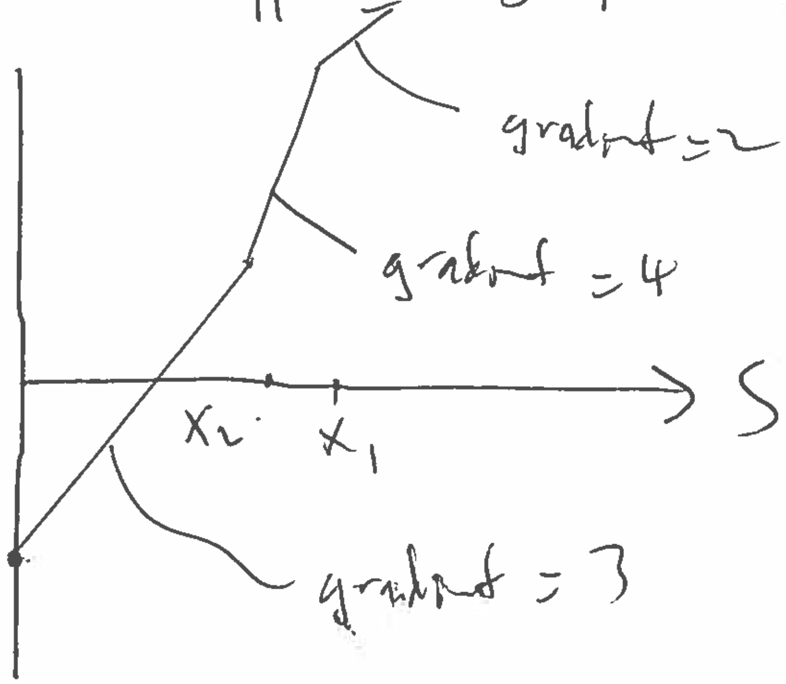
If $X_2 < S < X_1$

$\pi = S - 2(X_1 - S) + (S - X_2)$

$= 4S - 2X_1 - X_2$

If $X_1 < S$

$\pi = S + S - X_2 = 2S - X_2$



4

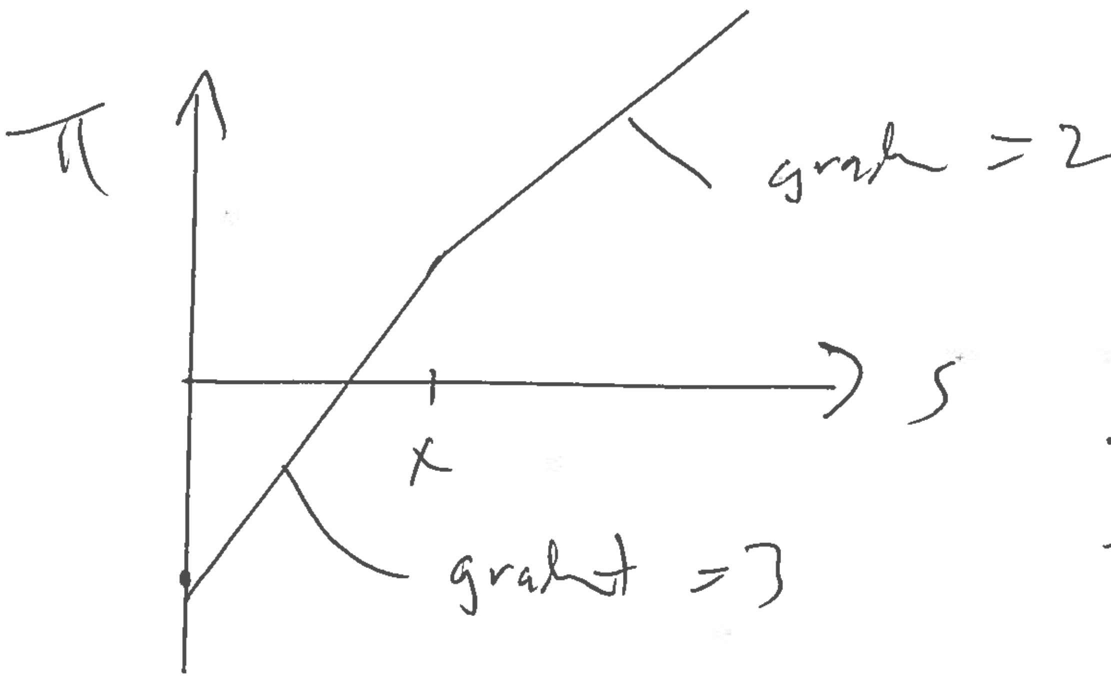
If $X_1 = X_2 = X$ B

If $0 < S < X$

$\pi = S - 2(X - S) = 3S - 2X$

If $X < S$

$\pi = S + (S - X) = 2S - X$



$\overline{\underline{4}}$

If $X_2 > X_1$

If $0 < S < X_1$

$\pi = S - 2(X_1 - S) = 3S - 2X_1$

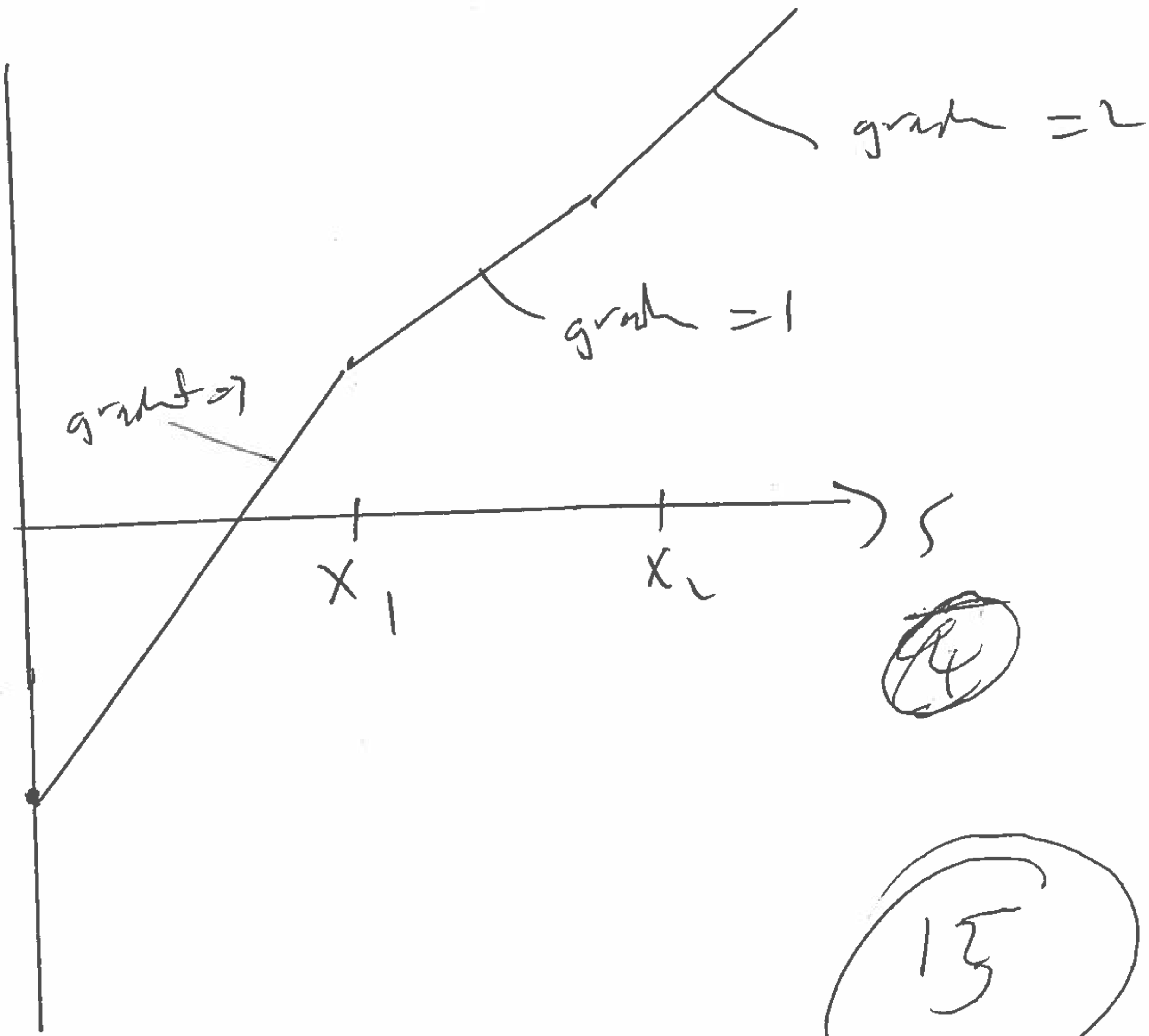
4

If $X_1 < S < X_2$

$\pi = 5$

If $S > X_2$

$\pi = S + S - X_2 = 2S - X_2$



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15

A4 1f $P = AS^{\alpha}$ [(i) seen similar (ii), (iii) unseen]

(i)

$$\frac{1}{2}\sigma^2\alpha(\alpha-1) + r\alpha - r = 0$$

$$\alpha^2\left(\frac{1}{2}\sigma^2\right) + \alpha\left(r - \frac{1}{2}\sigma^2\right) - r = 0$$

$$\alpha = 1,\ -2r/\sigma^2$$ 4

(ii)

On $S = S_f$

$$P(S=S_f) = e^{-\lambda S_f}$$ 2

$$\frac{dP}{dS}(S = S_f) = -\lambda e^{-\lambda S_f}$$ 2

smooth pasting

$$P(S \to \infty) \to 0$$ 2

(iii) $$A S_f^{\alpha} = e^{-\lambda S_f}$$ $\alpha = \frac{-2r}{\sigma^2}$

$$\alpha A S_f^{\alpha-1} = -\lambda e^{-\lambda S_f}$$

$$\frac{S_f}{\alpha} = -\frac{1}{\lambda} \Rightarrow S_f = -\frac{\alpha}{\lambda}$$

$$S_f = \frac{2r}{\sigma^2\lambda}$$ 5

15

BS (i). $dV = \frac{\partial V}{\partial S} dS + \frac{1}{2}\frac{\partial^2 V}{\partial S^2} dS^2$ 1

$+ \frac{\partial V}{\partial t} dt + \dots$

$E[dS^2] = \sigma^2 dt$ 2

$dV = \left[\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial S^2}\right] dt + \frac{\partial V}{\partial S} dS$

See simile

$\Pi = V - \Delta S$, $d\Pi = dV - \Delta dS$

Choose $\Delta = \frac{\partial V}{\partial S}$ 2

$d\Pi = \left[\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial S^2}\right] dt$ risky free

If no arbitrage $d\Pi = [r\Pi] dt$

$\Rightarrow \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial t^2} + rS\frac{\partial V}{\partial S} - rV = 0$

3

(ii) If $1 = \sigma S$

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2} S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

Unseen

If $V = S^2 f(t)$

$$f' + \sigma^2 f + 2rf - rf = 0$$

$$f' + (\sigma^2 + r) f = 0 \qquad f(T) = 1$$

$$f = \exp\left[\int_t^T (\sigma^2 + r)\, dt\right]$$

$$\underline{\underline{V = S^2 \exp\left[\int_t^T (\sigma^2 + r)\, dt\right]}}$$

(iii) $V = S^2 f_1(t) + f_2(t)$

Unseen

$$\frac{\partial V}{\partial t} + \frac{\sigma_1^2}{2} \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

$O(S^2)$ $\quad f_1' + 2rf_1 - rf_1 = 0$

$$f_1(T) = 1 \Rightarrow f_1 = \exp[r(T-t)]$$

$O(5^\circ)$ $\quad f_2' + \sigma_1^2 f_1 - r f_2 = 0$ $\quad \boxed{0}$

$$f_2 = A e^{rt} + \alpha e^{-rt}$$

$$-2r e^{-rt} \alpha = -\sigma_1^2 e^{r(T-t)}$$

$$\alpha = \frac{\sigma_1^2}{2r} e^{rT}$$

$$f_2 = A e^{rt} + \frac{\sigma_1^2}{2r} e^{r(T-t)}$$

Unseen

But $f_2(T) = 0$

$$A = -\frac{\sigma_1^2}{2r} e^{-rT}$$

$$f_2 = \frac{\sigma_1^2}{2r}\left[e^{r(T-t)} - e^{-r(T-t)}\right]$$

$\underline{\overline{8}}$

20

BL (i) If $V = e^{-D(T-t)} V_1(S,t)$ [?]

Seen in class

$$DV_1 + \frac{\partial V_1}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V_1}{\partial S^2} + (r-D)\frac{\partial V_1}{\partial S} - rV_1 = 0$$

$$\Rightarrow \frac{\partial V_1}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V_1}{\partial S^2} + (r-D)\frac{\partial V_1}{\partial S} - (r-D)V_1 = 0$$

$\Rightarrow$ BSE, with $r \rightarrow r-D$ $\frac{4}{}$

(ii)

$$C_d = e^{-D(T-t)} S N(d_{10}) - e^{-r(T-t)} N(d_{20})$$

(iii)

$$d_{10} = \frac{\log(S/X) + (r-D+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_{20} = \frac{\log(S/X) + (r-D-\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$\frac{4}{}$

(iii). ~~d_{10}~~ $T-t = 0.75$

$D = 0.05$

$\sigma = 0.2$

$r = 0.015$

$X = 51, \quad S = 50$

see similar

$d_{10} = -0.17928, \quad d_{20} = -0.352$

$N(d_{10}) = 0.42885$

$N(d_{20}) = 0.36227$

$Cd = 2.386295$

10 minutes

8

(iv). see similar

$C_d = C_1 e^{-D(T-t)}$

$P_d = P_1 e^{-D(T-t)}$

$C_1 + X e^{-(r-D)(T-t)} - P_1 - S = 0$

$e^{D(T-t)} C_d + X e^{-(r-D)(T-t)} - e^{D(T-t)} P_d - S = 0$

$C_d + X e^{-r(T-t)} - P_d - S e^{-D(T-t)} = 0$

4

20

1

B6 (i) $dS_i = \mu_i S_i \, dt + \sigma_i S_i \, dW_i$

$\rightarrow$ geometric Brownian motion

$\rightarrow$ changes in S_i linked to magnitude of S_i 2

$\rightarrow$ S_i cannot go negative 2

see st-ler

(ii) $d\Pi = dV - \Delta_1 dS_1 - \Delta_2 dS_2$

$$= \Big[\frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S_1} dS_1 + \frac{\partial V}{\partial S_2} dS_2 + \frac{1}{2} \frac{\partial^2 V}{\partial S_1^2} dS_1^2 + \frac{1}{2} \frac{\partial^2 V}{\partial S_2^2} dS_2^2 + \frac{\partial^2 V}{\partial S_1 \partial S_2} dS_1 dS_2 \Big]$$

$$- \Delta_1 [\mu_1 S_1 dt + \sigma_1 S_1 dW_1]$$
$$- \Delta_2 [\mu_2 S_2 dt + \sigma_2 S_2 dW_2]$$

Similar on ex. sheets

$$= \frac{\partial V}{\partial t}\, dt + \frac{\partial V}{\partial S_1}\left[\mu_1 S_1\, dt + \sigma_1 S_1\, dW_1\right] \quad (2$$

$$+ \frac{\partial V}{\partial S_2}\left[\mu_2 S_2\, dt + \sigma_2 S_2\, dW_2\right] + \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2}\, dt$$

$$+ \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2}\, dt + \sigma_1\sigma_2\rho S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2}\, dt$$

$$- \Delta_1\left[\mu_1 S_1\, dt + \sigma_1 S_1\, dW_1\right]$$

$$- \Delta_2\left[\mu_2 S_2\, dt + \sigma_2 S_2\, dW_2\right]$$

$$+ \quad o(dt)$$

similar to et sheets

$\Delta_i = \frac{\partial V}{\partial S_i}$ removes dW_i

(iii) $$d\Pi = \left[\frac{\partial V}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial S_1^2} S_1^2 \sigma_1^2 + \frac{1}{2}\frac{\partial^2 V}{\partial S_2^2} S_2^2 \sigma_2^2 + \frac{\partial^2 V}{\partial S_1 \partial S_2} S_1 S_2 \sigma_1 \sigma_2\right] dt$$

$$= r\Pi\, dt$$

$$= dt\, r\left[V - S_1 \frac{\partial V}{\partial S_1} - S_2 \frac{\partial V}{\partial S_2}\right]$$

$$\Rightarrow \frac{\partial V}{\partial t} + \frac{1}{2}\sigma_1^2 \frac{\partial^2 V}{\partial S_1^2} + \frac{1}{2}\sigma_2^2 \frac{\partial^2 V}{\partial S_2^2} + \rho\sigma_1\sigma_2 \frac{\partial^2 V}{\partial S_1 \partial S_2}$$

$$+ r S_1 \frac{\partial V}{\partial S_1} + r S_2 \frac{\partial V}{\partial S_2} - rV = 0$$

3

(iv) If $V = a_1 S_2 H(\xi, t)$

$$V(t=T) = \max(a_1 S_1 - a_2 S_2, 0)$$
$$= a_1 S_2 \max\left(\xi - \frac{a_2}{a_1}, 0\right)$$
$$\Rightarrow H(t=T) = \max(\xi - K, 0)$$

As $\xi \to \infty$, $H \to \xi$

As $\xi \to 0$, $H \to 0$

user (v)

$$\frac{\partial V}{\partial S_1} = a_1 S_2 \frac{\partial H}{\partial \xi}\left(\frac{1}{S_2}\right) = a_1 \frac{\partial H}{\partial \xi}$$

$$\frac{\partial V}{\partial S_2} = a_1 H - a_1 \frac{S_1}{S_2} \frac{\partial H}{\partial \xi}$$

$$\frac{\partial^2 V}{\partial S_1^2} = \frac{a_1}{S_2} \frac{\partial^2 H}{\partial \xi^2}$$

$$\frac{\partial^2 V}{\partial S_2^2} = \cancel{a_1 \frac{\partial H}{\partial \xi}\left(-\frac{S_1}{S_2^2}\right)} + \cancel{\frac{a_1 S_1}{S_2^2} \frac{\partial H}{\partial \xi}}$$
$$- a_1 \frac{S_1}{S_2} \frac{\partial^2 H}{\partial \xi^2}\left(-\frac{S_1}{S_2^2}\right) = \frac{\xi^2}{S_2^2} \frac{\partial^2 H}{\partial \xi^2}$$

$$\frac{1}{2}\sigma_1^2 \frac{q_1 S_1^2}{S_2}\frac{\partial^2 H}{\partial \xi^2} + \frac{1}{2} q_1 \sigma_2^2 \xi^2 \frac{S_1^2}{S_2}\frac{\partial^2 H}{\partial \xi^2}$$ 4

$$+ q_1 S_2 \frac{\partial H}{\partial t} + r S_1 q_1 \frac{\partial H}{\partial \xi}$$

$$+ r S_2\left[q_1 H - \frac{q_1 \xi}{S_2}\frac{\partial H}{\partial \xi}\right] - r q_1 S_1 H = 0$$

unseen

$$\frac{\partial H}{\partial t} + \left[\frac{1}{2}\sigma_1^2 + \frac{1}{2}\sigma_2^2\right]\xi^2 \frac{\partial^2 H}{\partial \xi^2} = 0$$

(i) – (iii) similar to material on example sheet. Remainder unseen.

5

20