MATH39032 (Mathematical modelling of finance): Examples 6

Extensions to Black Scholes: Lectures 13-14

1. Find a general solution to the Black-Scholes equation with known constant continuous dividend yield, D, for European call and put options when $V(S,t) = A(S)$

2. Calculate the value of a call on an asset (value S) that pays out *two* dividends $(S(t_{d_1}^-)d_1, S(t_{d_2}^-)d_2)$ at times (t_{d_1}, t_{d_2}) during the life of the option.

3. (i) Consider an asset of value S which pays just one dividend, Sd_y per share, during the lifetime of a put option, at $t = t_d$. In the absence of factors such as taxes and transaction costs, how is the value of the asset after payment $S(t_d^+)$ related to the value before payment $S(t_d^-)$? Justify your answer.

   (ii) How does the value of a put option on the asset, $P_d(S,t)$ change across the dividend date? Justify your answer.

   (iii) Using the above, compare the value of a European put option $P_d(S,t)$ on a single dividend-paying asset, with that of a non-dividend paying (vanilla) European put option $P(S,t,X)$, both with exercise price X. Consider the regimes $0 < t \leq t_d^-$ and $t_d^+ \leq t < T$ (the expiry date) separately. What effect does the dividend payment have on the value of the put?

   (iv) Suppose that the asset pays n dividends $Sd_1, Sd_2, \ldots, Sd_n$ per share during the lifetime of the option, at corresponding times $t_1, t_2, \ldots, , t_n$. What is the value of this dividend-paying option at time t, where t is between two dividend dates $(0 \leq t_{k-1} < t < t_k \leq t_n)$, compared with the corresponding vanilla European put option?