MATH39032 (Mathematical modelling of finance): Examples 1

Introduction: Lectures 1-3

1. What difference do you think the following would make to the price of a European call option? Explain your reasoning.

   (a) The strike price, X, being a lot lower than the current asset price S.

   (b) The strike price, X, being a lot higher than the current asset price S.

   (c) Increased volatility of the underlying asset.

2. An investor has bought a European call option from a Bank for \$2.50. The option is to buy IBM shares at a strike price of \$50 and is held until maturity. You may assume that the cost of borrowing is near zero ($r \approx 0$).

   (a) Under what circumstances would the investor exercise her option? Under what circumstances would she realise a profit? Draw a diagram illustrating the variation of her profit/loss with the stock price at maturity.

   (b) Draw an equivalent diagram for the Bank (the writer of the option) showing its profit/loss against the stock price at maturity.

   (c) Draw both these diagrams in the general case for a call option costing C with a strike price of X

3. A speculator has bought a European put option from a company for \$4. The option is to sell Vodafone shares at a strike price of \$60 and is held to maturity. You may assume that the cost of borrowing is near zero ($r \approx 0$).

   (a) Under what circumstances does the investor exercise his option? Under what circumstances would he realise a profit? Draw a diagram illustrating the variation of his profit/loss with the stock price at maturity.

   (b) Draw an equivalent diagram for the 'company' (the writer of the option) showing its profit/loss against the stock price at maturity.

   (c) Draw both these diagrams in the general case for a European put option costing P with a strike price of X.

4. (a) Show that for a forward contract, if the delivery price F is such that $F < Se^{rT}$ (where S is the current value of the underlying, r is the risk-free rate and T is the time to expiry) then there exists an arbitrage opportunity.

(b) If a stock has price S immediately before a dividend D is paid out, what is the price immediately after the payment? (Again attempt to use arbitrage arguments.)

5. This is a question on the valuation of bonds, $B(r, t)$.

   (i) Assuming an annual interest rate of 5% p.a., what is cost of a bond paying £100 in ten years time?

   (ii) If a bond with a maturity in ten years time pays \$1,000,000 with an annual interest rate of 5% for the first five years, and 6% for the remaining lifetime of the bond, what is the value of the bond (today)?

   Hint: this calculation will need to be broken up into two parts.

   (iii) A **coupon-bearing** bond pays **coupons** (i.e. dividends). Consider a coupon-bearing bond with a three-year maturity, which pays \$100 on maturity, together with a coupon of \$5 after one and two years. What is the value (today) of this bond? Assume an interest rate of 2.5% for the lifetime of the bond.

   Hint: coupons may be treated as 'mini-bonds' in their own right.