

# MATH39032

## (Mathematical modelling of finance):

### Examples 1

Introduction: Lectures 1-3

1. What difference do you think the following would make to the price of a European call option? Explain your reasoning.
  - (a) The strike price,  $X$ , being a lot lower than the current asset price  $S$ .
  - (b) The strike price,  $X$ , being a lot higher than the current asset price  $S$ .
  - (c) Increased volatility of the underlying asset.
2. An investor has bought a European call option from a Bank for \$2.50. The option is to buy IBM shares at a strike price of \$50 and is held until maturity. You may assume that the cost of borrowing is near zero ( $r \approx 0$ ).
  - (a) Under what circumstances would the investor exercise her option? Under what circumstances would she realise a profit? Draw a diagram illustrating the variation of her profit/loss with the stock price at maturity.
  - (b) Draw an equivalent diagram for the Bank (the writer of the option) showing its profit/loss against the stock price at maturity.
  - (c) Draw both these diagrams in the general case for a call option costing  $C$  with a strike price of  $X$
3. A speculator has bought a European put option from a company for \$4. The option is to sell Vodafone shares at a strike price of \$60 and is held to maturity. You may assume that the cost of borrowing is near zero ( $r \approx 0$ ).
  - (a) Under what circumstances does the investor exercise his option? Under what circumstances would he realise a profit? Draw a diagram illustrating the variation of his profit/loss with the stock price at maturity.
  - (b) Draw an equivalent diagram for the 'company' (the writer of the option) showing its profit/loss against the stock price at maturity.
  - (c) Draw both these diagrams in the general case for a European put option costing  $P$  with a strike price of  $X$ .
4. (a) Show that for a forward contract, if the delivery price  $F$  is such that  $F < Se^{rT}$  (where  $S$  is the current value of the underlying,  $r$  is the risk-free rate and  $T$  is the time to expiry) then there exists an arbitrage opportunity.

- (b) If a stock has price  $S$  immediately before a dividend  $D$  is paid out, what is the price immediately after the payment? (Again attempt to use arbitrage arguments.)
5. It is customary for shares in the UK to have prices between 100p and 1000p (in the US between \$10 and \$100). A company whose share price rises above this range will usually issue new shares to bring it back into range; this is called a **scrip issue** in the UK, and a **stock split** in the US. What is the effect of a two-for-one issue (i.e. two new shares for each old one) on the share price? How should option contracts be altered? What will be the effect on option prices?
6. This is a question on the valuation of bonds,  $B(r, t)$ .
- (i) Assuming an annual interest rate of 5% p.a., what is cost of a bond paying £100 in ten years time?
- (ii) If a bond with a maturity in ten years time pays \$1,000,000 with an annual interest rate of 5% for the first five years, and 6% for the remaining lifetime of the bond, what is the value of the bond (today)?  
*Hint: this calculation will need to be broken up into two parts.*
- (iii) A **coupon-bearing** bond pays **coupons** (i.e. dividends). Consider a coupon-bearing bond with a three-year maturity, which pays \$100 on maturity, together with a coupon of \$5 after one and two years. What is the value (today) of this bond? Assume an interest rate of 2.5% for the lifetime of the bond.  
*Hint: coupons may be treated as 'mini-bonds' in their own right.*