Lecture 6.

Rate of change of supersaturation

$$\frac{dS_i}{dt} = (S_i + 1)\left[\underbrace{\left(\frac{L_v}{c_p R_v T} - \frac{1}{R_a}\right)\frac{gw}{T}}_{\text{Adiabatic cooling \& pressure reduction}} - \underbrace{\left(\frac{1}{w_v} + \frac{L_v^2}{R_v T^2 c_p}\right)\frac{dw_c}{dt}}_{\substack{\text{reduction in } w_v \\ \text{due to condensation} \\ + \text{ increase in } T \\ \text{due to condensation}}} - \underbrace{\left(\frac{1}{w_v} + \frac{L_v L_s}{R_v T^2 c_p}\right)\frac{dw_i}{dt}}_{\substack{\text{reduction in } w_v \\ \text{due to sublimation} \\ + \text{ increase in } T \text{ due} \\ \text{to ice growth.}}}\right]$$

We can't solve analytically, but...

set $\frac{dS_i}{dt}$ to 0 [steady state]

We know previously that a single drop, mass m, radius a, grows from vapour:-

$$\frac{dm}{dt} = \frac{4\pi a S_l}{A_1} \qquad [\,kg\,s^{-1}\,]$$

and a single ice Xtal, mass m, capacitance C:

$$\frac{dm}{dt} = \frac{4\pi C S_i}{A_2} \qquad [\,kg\,s^{-1}\,]$$

Note that:

$$S_i = \frac{e}{e_{sat,i}} - 1$$

$$S_l = \frac{e}{e_{sat,l}} - 1$$

[e = vapour pressure, Pa
$e_{sat,l}$ = sat. vap. pressure w.r.t liquid, Pa
$e_{sat,i}$ = sat. vap. pressure w.r.t ice, Pa]

$$\therefore \quad S_i = (S_l + 1)\frac{e_{sat,l}}{e_{sat,i}} - 1$$

[Supersat w.r.t ice in terms of Supersat. w.r.t liquid]

To get the rate of change of mass mixing ratio, w_l or w_i multiply these growth rates by the number density of drops of Xtals.

$$\therefore \quad \frac{dw_l}{dt} = N_{drops} \times \frac{4\pi \bar{a}\, S_l}{A_1} \qquad \left[kg\, kg^{-1}\, s^{-1} \right]$$

$$\frac{dw_i}{dt} = N_{ice} \times \frac{4\pi \bar{c}}{A_2} \left[(S_l + 1) \frac{e_{sat,l}}{e_{sat,i}} - 1 \right]$$

S-S:

$$0 = \alpha w - \beta N_{drop} \bar{a} S_l - \gamma N_{ice} \bar{c} (S_l + \delta)$$

Rearrange for S_l :-

$$\boxed{S_l = \frac{\alpha w - \gamma N_{ice} \bar{c} \delta}{(\beta N_{drop} \bar{a} + \gamma N_{ice} \bar{c})}} \qquad \left[\text{Quasi-steady supersat.} \right]$$

Example

1. $T = 258\,k$, $P = 500\,hPa = 500 \times 10^2\,Pa$

Cloud with $w = 1\,ms^{-1}$, $N_{drops} = 100 \times 10^6\,kg^{-1}$, $\bar{a} = 30 \times 10^{-6}\,m$

$$\alpha = 6.6 \times 10^{-4}\ m^{-1}$$

$$\beta = 3.1 \times 10^{-4}\ kg\,m^{-1}s^{-1}$$

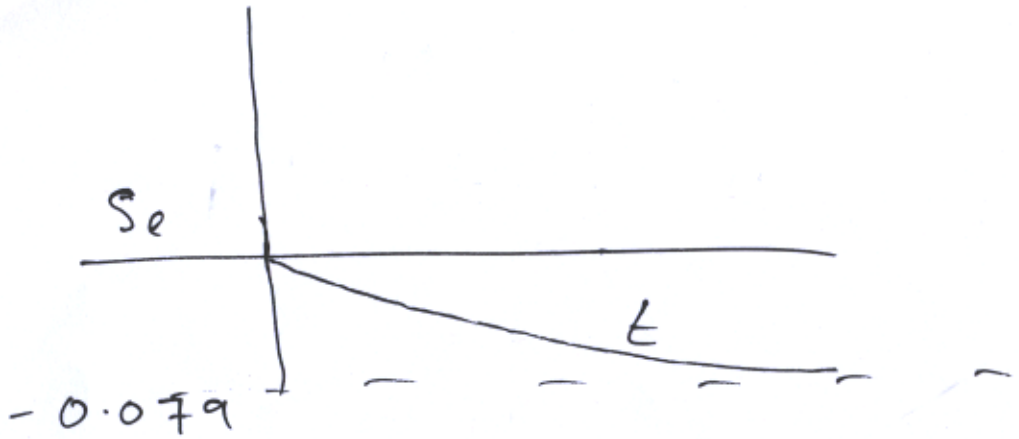
$$\gamma = 3.1 \times 10^{-4}\ kg\,m^{-1}s^{-1}$$

$$\delta = 0.1375$$

a) Steady supersaturation? $S_\ell = \dfrac{\alpha w}{\beta N_{drop} \bar{a}} = 7 \times 10^{-4}$.

b) $N_{ice} = 10^6\ kg^{-1}$, $\bar{c} = 200 \times 10^{-6}\,m$

$$S_\ell = \frac{\alpha w - \gamma N_{ice} \bar{c} \delta}{\beta N_{drop} \bar{a} + \gamma N_{ice} \bar{c}} = -0.079$$ [cloud glaciates]



c) Increase $w = 25 ms^{-1}$

$S_l = 0.008$ [cloud does not glaciate]

d) $w = 0 ms^{-1}$, $N_{drop} = 0$

$S_l = -\delta$

$= -0.1375$ [this is ice saturation].