A.7 Lecture 9 and 10

The continuous growth model again can be solved analytically so could in principle show up on the exam. You would be given Equation 4.8 and asked to worked from there.

Example 4.7 Explain why with reference to the solutions of the relevant growth equations why growth by vapour diffusion leads to a narrowing of the distribution, whereas growth by collision and coalescence leads to a broadening.

Answer See example 2.4c to explain why growth from the vapour leads to narrowing of the distribution. For collision-coalescence:

- The solution to the growth equation is (Equation 4.12): $D(t) = \left(\frac{\bar{E}w_l a(1-b)}{2\rho_w}t + D_0^{1-b}\right)^{\frac{1}{1-b}}$.
- Or rearranging with $b = 2$: $D(t)^{-1} = \frac{\bar{E}w_l a(1-b)}{2\rho_w}t + D_0^{-1}$.
- So if we consider two drops starting at different initial sizes, D_1 and D_0, the difference in the reciprical of their sizes with time is: $D_1(t)^{-1} - D_0(t)^{-1} = D_1^{-1} + D_0^{-1}$. If the difference between the reciprical of their sizes is a constant with time then the difference in size gets larger with time (if they are growing).

Example 4.7a Assuming that the collision efficiency is equal to 1, the liquid water mixing ratio is 1×10^{-3}kg kg^{-1} and the intial diameter is $D_0 = 10\,\mu$m what will the diameter be after 6350 seconds?

Answer Use Equation 4.12: $D(t) = \left(\frac{\bar{E}w_l a(1-b)}{2\rho_w}t + D_0^{1-b}\right)^{\frac{1}{1-b}}$ with $b = 2$ and $a = 3.129 \times 10^7$, to get that $D(t = 1000) \cong 1.5$ mm. By contrast, growth at 2% supersaturation at 900 hPa gives 117 μm radius, or 234 μm diameter (see Example 2.4) for details on how to do the growth from the vapour calculation.

Example 4.7b The rain-drop size distribution is exponentially distributed and has an intercept of n_0 and a slope parameter of λ_0. If the terminal velocity of the rain drops can be described as a power law: $v_t = aD^b$ derive an expression for the mass flux of precipitation.

Answer The flux of precipitation is the integral of the product of the mass and fall speed: $\chi_f = \int aD^b \frac{\pi\rho_w}{6} D^3 n_0 \exp(-\lambda_0 D)\, dD$. By noting the integrals of modified gamma functions (previous lecture) we can write this as:

$$\chi_f = \frac{a\pi\rho_w n_0 \Gamma(4+b)}{6\lambda_0^{4+b}}$$

The heat budget of a hail-stone can have important consequences in cloud microphysics and crops up in several areas (e.g. thunderstorm electrification, rime-splintering). I won't ask you to solve the heat budget equation on the exam, but I could ask you to explain what determines the temperature of a riming hail-stone. Make sure you can do this.

Example 4.8 Explain the processes responsible for transferring heat to a hail-stone that is riming.

Answer With reference to Equation 4.15:

- The accreted water at the ambient cloud temperature $T > 0°C$ should approach the triple point before it can freeze. This requires heat to be transfered to / from the liquid water.
- Once frozen the rime should approach the temperature of the hail-stone, which requires heat to be transfered away from the rime. Ice has a difference heat capacity than liquid water though.
- Note if the rime water is already colder than 0°C then it doesn't have to freeze at the triple point, but the amount of heat required for the ice to approach the temperature of the hail stone is equivalent to the specific heat of liquid water approaching the triple point and then ice water approaching the hail temperature (by conservation of energy).
- The latent heat of fusion of the accreted water is released once it freezes, heating the hail stone.
- The latent heat of sublimation is released as the hail-stone grows from the vapour.

Example 4.8a What is the splinter production rate due to the H–M process if a graupel pellet diameter 0.5 mm falls through a supercooled liquid cloud of $w_l = 1 \times 10^{-3}$ kg/kg at −5°C?

Answer Use Equations 4.14 and 4.17: $\frac{\pi}{4}D^2 aD^b w_l \times 350 \times 10^6$, with $a = 140$ and $b = 0.5$. Thus $\frac{\pi}{4}\left(0.5 \times 10^{-3}\right)^2 \times 140 \left(0.5 \times 10^{-3}\right)^{0.5} \times 1 \times 10^{-3} \times 350 \times 10^6 \cong 0.2$ particles s^{-1}.

Example 4.8b Explain why larger hail-stones have a surface temperature that is elevated more than smaller hail-stones.

Answer Because they sweep out a larger volume of air per second (due to their larger size and larger fall speed). Hence they collect more liquid water per second and because this water freezes it releases more heat by the latent heat of fusion.