Topic 4
Growth of precipitation particles

We considered the growth of droplets from the vapour in previous lectures. From the theory we saw that droplets of different sizes growing by vapour diffusion tend to get closer together in size with time. Therefore, after a some time this theory would predict that the droplet size-distribution in a cloud becomes very narrow. We know intuitively that clouds rain, but not all of the cloud falls as rain—some drops remain in the cloud. This must mean that growth by vapour diffusion is not the only growth process in a cloud. We will finish off our study of cloud microphysics by studying processes which can broaden the size distribution in clouds.

4.1 Collision and coalescence

Collision and coalescence is the name given to the process where drops collide with other drops and coalesce within a cloud. You may also hear to it referred to as warm rain autoconversion; however, this is modellers jargon.

For typical drops it can be shown that Brownian motion is not a large factor governing their motion. The motion occurs primarily due to gravitational settling, which is governed by Newton's second law of motion:

$$\frac{1}{2}\rho_a u^2 \frac{\pi}{4} D^2 C_d\left(Re\right) - \frac{\pi\left(\rho_w - \rho_a\right)}{6} D^3 g = \frac{\pi \rho_w}{6} D^3 \ddot{z} \tag{4.1}$$

where $\ddot{z}$ is measured upwards. For Stokes flow, (i.e. non-turbulent) $C_D(Re) = 24/Re = \frac{24\eta}{uD\rho_a}$. Here η is the visocity of air, which depends on temperature, but can be taken to be 1.718×10^{-5} Pa s for this course.

In practice we can assume that the drop is falling at it's terminal velocity. Other researchers have found that turbulent air motions that are often present in clouds can enhance the collision of drops over that expected by pure gravitational settling by perhaps a factor of 2, but it depends on the strength of the turbulence. This arises because the drops have inertia so do not necessarily follow the air streamlines, neglecting these difficulties and assuming the acc^n is zero we have:

$$\frac{24}{8} u \pi D \eta = \frac{\pi\left(\rho_w - \rho_a\right)}{6} D^3 g \tag{4.2}$$

Rearranging for the terminal velocity, u yields:

$$u = \frac{\left(\rho_w - \rho_a\right)}{18\eta} g D^2 = a D^b = 3.129 \times 10^7 D^2 \tag{4.3}$$

so the terminal velocity can be parametrised by a power law in diameter.

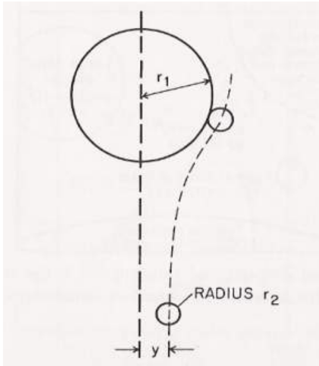

Figure 4.1: A small drop in the path of a large drop will not necessarily collide with it due to inertial and aerodynamic forces.

4.1.1 Collision efficiency

As large drops fall past small drops the small drops may be swept up by the large drops. The air will move around the drop, which causes an acceleration of the air perpendicular to the direction the drop is falling in. This influences the motion of the small drop relative to the large drop, thus moving it away from the large drop. There is a displacement of separation, y, (measured from the two droplet centres) above which the two drops will move far apart enough so that they will not collide. The collision efficiency is defined as the ratio of the total area that would result in a collision πy^2 to the geometrical area of overlap, $\pi (r_1 + r_2)^2$.

$$E(r_1, r_2) = \frac{y^2}{(r_1 + r_2)^2} \tag{4.4}$$

$$E(D, d) = \frac{4y^2}{(D + d)^2} \tag{4.5}$$

$E(D, d)$ is small when the collected drop d is small compared to the collector D. Once the collected drop size is larger than about $10\,\mu$m it can be assumed that $E(D, d) \cong 1$ although this is a generalisation.

4.1.2 Sweep-out kernel

The volume of air swept out per second by the large drop in a reference frame that sets the velocity of the small drop to zero is

$$\frac{\pi}{4}(D + d)^2 \left|u(D) - u(d)\right| \tag{4.6}$$

This is also called the sweep-out kernel.

4.1.3 Continuous growth model

This model considers the growth of a drop that has somehow managed to find itself slightly bigger than the rest of the other drops probably by heterogeneities in the atmosphere or differences in the aerosol size distribution on which the drops grew in the first place. We assume the drops grow continuously in size by collisions with other drops so the number of drops swept out per second by the large drop is:

$$\frac{\pi}{4}(D+d)^2\,|u(D)-u(d)|E(D,d)\frac{dN}{dd}dd \tag{4.7}$$

where $\frac{dN}{dd}$ is the drop size distribution function (see earlier lectures). Therefore the volume growth rate of the large drop is calculated by recognising that each addition of a drop size d adds a volume of $\frac{\pi}{6}d^3$ to the large drop:

$$\frac{dV}{dt} = \int_0^D \frac{\pi}{6}d^3\frac{\pi}{4}(D+d)^2\,|u(D)-u(d)|E(D,d)\frac{dN}{dd}dd \tag{4.8}$$

Making a change of variable $V=\frac{\pi}{6}D^3$ and $\therefore \frac{dV}{dt}=\frac{\pi}{2}D^2\frac{dD}{dt}$ enables us to write down the diameter growth rate:

$$\frac{dD}{dt} = \int_0^D \frac{1}{6}d^3\frac{\pi}{2}\frac{(D+d)^2}{D^2}|u(D)-u(d)|E(D,d)\frac{dN}{dd}dd \tag{4.9}$$

We could substitute an exponential form for $\frac{dN}{dd}$ and integrate; however, it is easier to approximate and say that $D+d\cong D$ and that $u(d)\cong 0$ so that Equation 4.9 becomes:

$$\frac{dD}{dt} \cong \int_0^D \frac{1}{6}d^3\frac{\pi}{2}u(D)E(D,d)\frac{dN}{dd}dd \tag{4.10}$$

Note that $\int_0^D \frac{\pi}{6}\rho_w d^3\frac{dN}{dd}dd$ would just be the liquid water content, w_l so integrating the RHS of Equation 4.10:

$$\frac{dD}{dt} \cong \frac{\bar{E}w_l a D^b}{2\rho_w} \tag{4.11}$$

where w_l is the liquid water mixing ratio. This has solution:

$$D(t) = \left(\frac{\bar{E}w_l a(1-b)}{2\rho_w}t + D_0^{1-b}\right)^{\frac{1}{1-b}} \tag{4.12}$$

Please note that the continuous growth equation is an approximation because in reality growth occurs by discrete events of droplet capture and the radius cannot take on any value, but it has to be multiples of values. In practice this leads to even quicker broadening, but we wont cover the relevant equations here—they are more the scope of a PhD.

However, we still have difficulty in explaining the observations that rain size distributions falling from clouds are exponentially distributed—where do the small particles come from if they are constantly growing? The answer is from drop break-up.

4.2 Coalescence and drop breakup

Not all collisions between drops will result in a coalescence event. Sometimes drops will collide and bounce off each other because the surface tension force is quite high compared to the momentum of the drops. This is treated by multiplying Expression 4.7 by another efficiency, the coalescence efficiency, E_{coal}. For this coarse we will not treat this explicitly.

Sometimes collisions between drops will cause the resulting drop to become unstable and breakup—so called collision induced breakup. Once droplets grow to very large sizes ($\cong$ 6mm in diameter) the surface tension of water is insufficient to hold the drop together and they can break-up spontaneously—so called spontaneous raindrop breakup.

There are different 'modes' of breakup:

Neck / filament originates through glancing blows and is where a filament stretches away from the bulk of the drop and detaches.

Sheet originates when the large drop is split into two as a droplet collides slightly off centre with it. Half of the large drop is torn off in a sheet.

Disk originates when the smaller drop collides at the centre of the larger drop. Coalescence occurs temporarily and the drop spreads out into a sheet before disintegrating.

The many 'small' particles in the distribution (which we have said is negative exponential) arise due to droplet break-up. The physics governing this process is beyond the scope of this course, but the final result is that equilibrium size distributions can be approached where there is a dynamic equilibrium between the growth by collision-coalescence and breakup.

4.3 Accretion and aggregation

Analogous equations to Equation 4.12 can be written down to *approximately* describe the growth of ice by accretion of liquid drops and also the aggregation of ice crystals. For accretion there is the added complexity that when droplets freeze onto the ice particles they release latent heat and warm the surface—this can cause the ice crystal to melt or sublime. For aggregation of ice crystals the geometry of the crystals complicates things, but generally the principle is the same. For aggregation the collision efficiency and coalescence efficiency are grouped into one efficiency which we call the aggregation efficiency, $E_{agg} = E \times E_{coal}$ and this is approximately equal to 0.1.

Example 4.6 *Explain why with reference to the solutions of the relevant growth equations why growth by vapour diffusion leads to a narrowing of the distribution, whereas growth by collision and coalescence leads to a broadening.*

Other questions to consider:

- Assuming that the collision efficiency is equal to 1, the liquid water mixing ratio is 1×10^{-3}kg kg^{-1} and the intial diameter is $D_0 = 10\,\mu$m what will the diameter be after 1000 seconds?
- How does this compare to growth at 2% supersaturation in a cloud at $T = 283.15$K?
- Understand the reason exponential rain drop spectra are maintained.

THE KEY POINTS TO TAKE HOME HERE ARE:

- Be able to derive / apply the continuous growth equation.
- Understand the concept of sweep out, collision efficiency and gravitational kernel.

4.4 Hail growth in a cloud

The first cloud physics conference I attended Roland List, the secretary general of IAMAS gave a talk on hail growth within clouds. The talk was about why hail-stones falling through the air precess like a spinning top, but Roland went into lots of detail on how they grow. At the end of the talk we were all puzzled and so a question was asked: "so why do hail-stones precess as they fall?" Roland looked up to the audience and said, "because they do!".

4.4.1 Growth by accretion / riming

Hail-stones or hail-embryos, which are called *graupel* pellets, grow in a cloud by vapour diffusion, aggregation and accretion of liquid water or *riming* as cloud physicists call it. We have already covered the physics of growth of ice from the vapour and the continuous growth model (which can be applied with limitations to consider aggregation and riming) so why do we need to have a separate section for hail growth? Well it turns out that when these growth processes are considered together there are some noteworthy differences.

Firstly we can multiply Equation 4.8 by the density of water to get the mass growth rate of a hail stone:

$$\frac{dm_{rime}}{dt} = \rho_w \int_0^D \frac{\pi}{6} d^3 \frac{\pi}{4} (D+d)^2 \left|u(D) - u(d)\right| E(D,d) \frac{dN}{dd} dd \tag{4.13}$$

where m_{rime} is the mass of rime accreted and again make the assumption that $D+d \cong D$ and $u(D) - u(d) \cong u(D)$ to find that:

$$\frac{dm_{rime}}{dt} = \frac{\pi}{4} D^2 a D^b w_l \tag{4.14}$$

where w_l is either the liquid water mixing ratio or water content and $a = 140$ and $b = 0.5$ in SI units.

4.4.2 Heat balance of a hail /graupel particle

When hail-stones grow by riming, the liquid water that is accreted onto the drops may freeze and as it does it releases the latent heat of fusion, $L_f \cong 3.12 \times 10^5$ J kg^{-1}, thus warming the surface of the particle. This reduces the rate of growth of the particle from the vapour, why?

During heavy riming the surface temperature of the particle can warm to the melting point of water so that the particle will start to melt. Again this changes the rate of growth of the particle from the vapour, why?

We make the assumption that the hail-stone temperature is in steady-state and that the supply of heat to the hail-stone is conducted away (Fourier's law). Therefore:

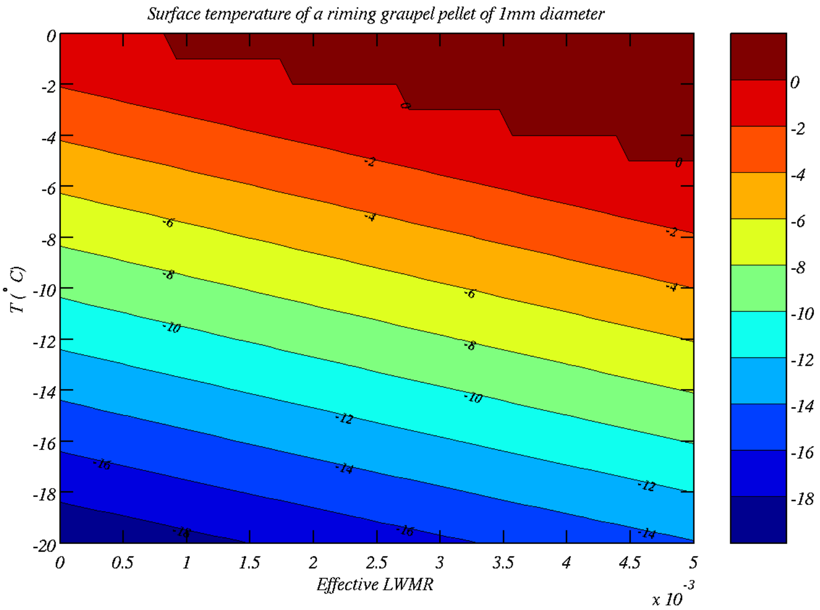

Figure 4.2: Surface temperature of a 1mm graupel pellett growing by riming and vapour diffusion.

$$4\pi ak\left(T_a - T_\infty\right) = L_s\frac{dm_{diff}}{dt} + c_w\left(T_\infty - T_0\right)\frac{dm_{rime}}{dt} + \\ fL_f\frac{dm_{rime}}{dt} + c_i f\left(T_a - T_0\right)\frac{dm_{rime}}{dt} - L_f\frac{dm_{melt}}{dt} \quad (4.15)$$

where $T_0 = 273.15$K and c_w and c_i are the specific heat capacities of water and ice.

At first glance it seems like this equation could be solved for the temperature of the particle, T_a, but the growth rate from the vapour depends on the surface temperature too, $\frac{dm_{diff}}{dt} = \frac{4\pi Da}{R_v}\left(\frac{e}{T_\infty} - \frac{e_s(T_a)}{T_a}\right)$, so the whole thing has to be solved iteratively.

The result is that the surface temperature of a riming hail-stone depends on the liquid water content in cloud. If we assume that the hail-stone is in a mixed-phase cloud then the ambient vapour pressure will be water saturated. Figure 4.2 shows the surface temperature of a hail stone growing by riming.

What do you think will happen under extremely heavy riming in a cloud with ambient temperature lower than the melting point of water? Will the graupel particle melt? Will it sublimate?

4.5 Ice multiplication by rime-splintering

The Hallett–Mossop (H–M) process is the name given to the process that occurs when hail or graupel pellets rime in the temperature region $-7.5 < T < -2.5$°C.

In this temperature regime the freezing pattern of drops on to the hail is such that the drops fracture as they freeze and emit copious ice 'splinters'. The number of splinters produced depends on the temperature, being a maximum at −5°C. The rate of splinter production is roughly 350 mg^{-1} of rime accreted and tails off linearly to zero 2.5 degrees either side of −5°C.

$$f_{H-M}(T) \cong \begin{cases} (2.5 - T)/2.5 & \text{for} - 5.0 < T < -2.5 \\ (7.5 + T)/2.5 & \text{for} - 7.5 < T < -5.0 \\ 0 & \text{otherwise} \end{cases} \tag{4.16}$$

The number of splinters generated as one hail-stone accretes liquid water therefore follows from Equation 4.14:

$$P_{H-M} = 350 \times 10^6 f_{H-M}(T) \frac{dm_{rime}}{dt} \tag{4.17}$$

where P_{H-M} is the production rate of ice due to H–M multiplication.

The reason for the temperature dependence is thought to be that:

At high T The droplets take longer to freeze, so tend to flatten onto the graupel pellet before freezing.

In the H–M zone The droplets come in contact with the ice surface on one side and there is dendritic ice growth (crystallisation) through the liquid water which eventually contacts the other side of the droplet this causes an ice shell to form around the drop and liquid water to remain trapped in the ice shell. The liquid water in the shell freezes as an ice front propagates through the drop and because ice is less dense than liquid water there is a build up in pressure within the drop, which eventually causes the frozen drop to fracture, ejecting splinters.

At low T The ice front propagates through the drop quicker than the ice shell can form.

4.5.1 Dependence of the surface temperature of hail on the rime-splinter process?

As we have seen the temperature of riming graupel changes with w_l. Figure 4.3 shows this for different size graupel particles. So rime-splintering could occur at many temperatures in a cloud. Why is this?

4.5.2 Rime-splintering as an exponential growth process

Ice crystals ejected by the H–M process can be collected by rain-drops and cause them to freeze. This generates new graupel / hail-stones which can act as new 'rimers' in the H–M zone.

4.5.3 Observations of ice in cumulus clouds

The number of measured ice crystals in cumulus (Cu) frequently exceeds the number of measured IN with an ice nucleus counter by several orders of magnitude. In many of these instances it seems that the H–M process can explain the discrepancy.

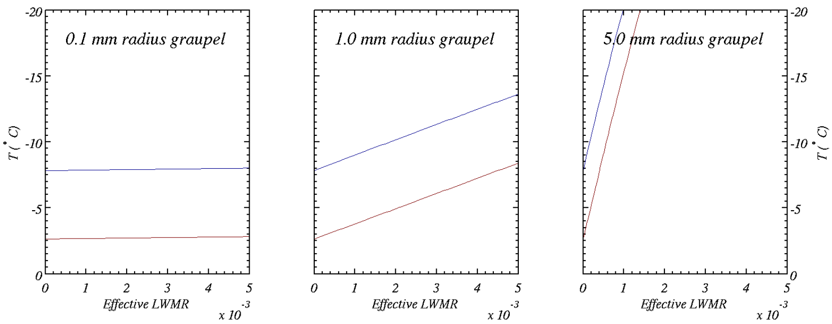

Figure 4.3: Plots showing the bounds of the H–M process for hail-stones of different sizes in T-effective LWMR space.

Example 4.7 *Explain the processes responsible for transferring heat to a hail-stone that is riming.*

Other questions to consider:

- What is the splinter production rate due to the H–M process if a graupel pellet diameter 0.5 mm falls through a supercooled liquid cloud of $w_l = 1 \times 10^{-3}$ kg/kg at -5°C?
- Explain why larger hail-stones have a surface temperature that is elevated more than smaller hail-stones.

THE KEY POINTS TO TAKE HOME HERE ARE:

- Understand and explain the H–M process and associated equations.
- Understand the terms in the heat balance of hail-stones.