A.6 Lecture 7 and 8

The topic on supersaturation in clouds brings together the droplet and ice crystal growth equations, the hydrostatic relation, and the first law of thermodynamics (which is used to derive θ and $\theta_{q,sat}$). It therefore brings together everything done so far and is a very important concept in cloud physics. Reading through and answering the descriptive examples in the notes will help you for the exam.

Example 3.7 Explain what all the terms in Equation 3.7 mean.

Answer see notes.

Example 3.7a For an ascent rate of 2 m s^{-1}, what is the steady-state supersaturation in a liquid only cloud containing drops of concentration 1000 mg^{-1} of air, radius 10 μm at a pressure of 900 hPa and temperature of 273 K?

Answer Use $S_l = \frac{a_0 w}{b_l N_l a_l}$ where $a_0 = \frac{g}{c_p} - \frac{g}{R_a T} \sim 9.63 \times 10^{-3}$, $N_l = 1000 \times 10^6$ kg^{-1}, $a_l =$ 10 μm and b_l is the thermodynamic factor $4\pi \frac{\left(\frac{1}{w_v} + \frac{L_v^2}{R_v T^2 c_p}\right)}{\frac{R_v T}{e_{s,liq}(T) D_v} + \frac{L_v}{Tk}\left(\frac{L_v}{R_v T} - 1\right)} \sim 3.1 \times 10^{-4}$. The water vapour mixing ratio w_v can be assumed to be at water saturation so $w_v \cong \frac{\epsilon e_{s,liq}(T)}{P} \sim 4.18 \times 10^{-3}$ kg/kg and $e_{s,liq}(T) \sim 605$ Pa is defined from the saturation vapour pressure equation, D_v and k were defined in previous lectures $D_v \sim 2.11 \times 10^{-5}$ and $k \sim 0.025$. Taking all this together gives:

$$S_l \cong \frac{9.63 \times 10^{-3} \times 2}{2.45 \times 10^{-5} \times 1000 \times 10^6 \times 10^{-6}} \cong 0.006$$

Example 3.7b In a mixed phase cloud, what is the ascent rate required to maintain supercooled liquid water at -15°C and 700 hPa if the ice crystal concentration is 1 mg^{-1} of air and the ice crystal average radii are 100 μm?

Answer Use $w = \frac{b_{1,i} N_i a_i}{a_0}$ where $a_0 = \frac{g}{c_p} - \frac{g}{R_a T} \sim 9.62 \times 10^{-3}$, $N_i = 1 \times 10^6$ kg^{-1}, $a_i =$ 100 μm and $b_{1,i}$ is the thermodynamic factor $4\pi \left(\frac{e_{sat,l}}{e_{sat,i}} - 1\right) \frac{\left(\frac{1}{w_v} + \frac{L_v L_s}{R_v T^2 c_p}\right)}{\frac{R_v T}{e_{s,ice}(T) D_v} + \frac{L_s}{Tk}\left(\frac{L_s}{R_v T} - 1\right)} \sim 4.45 \times 10^{-5}$. The water vapour mixing ratio w_v can be assumed to be at water saturation so $w_v \cong \frac{\epsilon e_{s,liq}(T)}{P} \sim 1.3 \times 10^{-3}$ kg/kg and $e_{s,liq}(T) \sim 189$ Pa and $e_{s,ice}(T) \sim 163$ are defined from the saturation vapour pressure equations, D_v and k were defined in previous lectures $D_v \sim 2.11 \times 10^{-5}$ and $k \sim 0.025$. Taking all this together gives:

$$w \cong \frac{4.45 \times 10^{-5} \times 1 \times 10^6 \times 100^{-6}}{9.63 \times 10^{-3}} \cong 0.45 \text{ m s}^{-1}$$