Topic 3
Supersaturation in clouds

So far when considering the water content within a cloud we have dealt with the bulk thermodynamical considerations within a cloud so when saturation is reached we assumed condensation occured instantaneously. This theory works reasonably well for describing the water content within liquid clouds or even ice clouds, but cannot be applied to mixed-phase clouds. In real clouds the supersaturation is vitally important to the outcome of the cloud and the amount of precipitation it produces because it affects the number of cloud drops and ice crystals that are in the cloud and therefore affects the production of precipitation.

The supersaturation in clouds is still an active area of research (one that I focus on quite a bit), supersaturation is something that is extraordinarily difficult to measure using an aircraft, due to its transient nature, and something that is *apparently* difficult to include within UK Met.Office (Met.Office) models and climate models.

Why do we need to know about supersaturation?

- Because we need to be able to predict the activation of cloud drops and the nucleation of ice crystals by heterogeneous deposition (which depends on supersaturation)
- Because we need to predict the growth of particles from the vapour (which depends on supersaturation)
- Because we need to know how long liquid water persists in a cloud (Bergeron-Findeison process).

3.1 Rate of change of supersaturation in a cloud

A theory describing the supersaturation in a cloud can be derived from definitions we have covered previously. Firstly as previously described the *supersaturation* is given by $S_l = \frac{(e-e_{sat,l})}{e_{sat,l}}$ so the rate of change is (quotient rule):

$$\frac{dS_l}{dt} = \frac{1}{e_{sat,l}}\frac{de}{dt} - \frac{e}{e_{sat,l}^2}\frac{de_{sat,l}}{dt} \tag{3.1}$$

From the previous definition of vapour mixing ratio $w_v = \frac{\epsilon e}{P}$ we can rearrange ($e = \frac{w_v P}{\epsilon}$) and define the rate of change of vapour pressure (product rule):

$$\frac{de}{dt} = \frac{1}{\epsilon}P\frac{dw_v}{dt} + \frac{1}{\epsilon}w_v\frac{dP}{dt} \tag{3.2}$$

Now we can use the Clausius-Clapyeron equation:

$$\frac{de_{sat,l}}{dt} = \frac{de_{sat,l}}{dT}\frac{dT}{dt} = \frac{L_v e_{sat}}{R_v T^2}\frac{dT}{dt} \tag{3.3}$$

and the 1st law of thermodynamics (time derivative):

$$c_p \frac{dT}{dt} - R_a \frac{T}{P}\frac{dP}{dt} - L_v \frac{dw_l}{dt} - L_s \frac{dw_i}{dt} = 0 \tag{3.4}$$

and substitute these into Equation 3.1 to give us:

$$\frac{dS_l}{dt} = \frac{1}{e_{sat,l}}\left(\frac{1}{\epsilon}P\frac{dw_v}{dt} + \frac{1}{\epsilon}w_v\frac{dP}{dt}\right) - \frac{e}{e_{sat,l}}\frac{L_v}{R_v T^2}\left(\frac{R_a T}{c_p P}\frac{dP}{dt} + \frac{L_v}{c_p}\frac{dw_l}{dt} + \frac{L_s}{c_p}\frac{dw_i}{dt}\right) \tag{3.5}$$

Following this we can take the time derivative of the hydrostatic relation for the parcel:

$$\frac{dP}{dt} = -\frac{gP}{R_a T}w \tag{3.6}$$

(where w is the vertical wind) and substitute Equation 3.6into Equation 3.5:

$$\frac{dS_l}{dt} = (S_l + 1)\left[\left(\frac{g}{c_p} - \frac{g}{R_a T}\right)w - \left(\frac{1}{w_v} + \frac{L_v^2}{R_v T^2 c_p}\right)\frac{dw_l}{dt} - \left(\frac{1}{w_v} + \frac{L_v L_s}{R_v T^2 c_p}\right)\frac{dw_i}{dt}\right] \tag{3.7}$$

where we have also used the fact that total water in the parcel is conserved:

$$\frac{dw_v}{dt} + \frac{dw_l}{dt} + \frac{dw_i}{dt} = 0$$

Recall the growth rates of drops and ice crystals $\frac{dm}{dt} \cong \frac{4\pi a S_l}{A}$, so that the total rate of change of mass (liquid or ice) is $\frac{dw_{l,i}}{dt} = N_{l,i}\frac{dm_{l,i}}{dt}$ and that $w_v = \frac{\epsilon e}{P}$. Thus Equation 3.7 describes the rate of change of supersaturation in a parcel of cloudy air with liquid drops and ice crystals.

The terms in Equation 3.7 are as follows:

Left hand side: This is the rate of change of supersaturation.

First term on right: The positive term describes how the saturation ratio increases due to the decrease in temperature of the parcel (conservation of energy), while the negative term describes the change due to the change in pressure of the air, which changes the vapour pressure of water vapour (see Equation 3.2).

Second term on right: This is a sum of terms describing the reduction of water vapour as it condenses onto liquid water and also the reduction in supersaturation due to latent heating of the air (latent heat release and then conduction to the air).

Third term on right: This is a sum of terms describing the reduction of water vapour as it condenses onto ice water and also the reduction in supersaturation due to latent heating of the air (latent heat release and then conduction to the air).

Example 3.5 *Explain what all the terms in Equation 3.7 mean.*

THE KEY POINTS TO TAKE HOME HERE ARE:

- Be able to explain the different terms in Equation 3.7.