2.5 Growth of single ice crystals

Recall that the electrostatic potential function, Φ ($\bar{E} = -\nabla\Phi$) satisfies Laplace's equation, $\nabla^2\Phi = 0$ and that Φ_S is a constant on a conductor and Φ_∞ is a constant at ∞. If we assume a growing or sublimating ice crystal of the same geometry as a conducting body, in the steady-state ρ_v also satisfies Laplace's equation.

Recall Gauss' law from electro-statics, that the flux of $\bar{E}$ through a closed surface is equal to the charge enclosed within the surface (remember $Q = CV$):

$$\int_S \nabla\Phi \cdot \hat{n} dS = -\frac{Q}{\epsilon_0} = -\frac{1}{\epsilon_0} C_e \left(\Phi_S - \Phi_\infty\right) \tag{2.45}$$

Therefore we can draw a complete analogy with the growth of a crystal:

$$\frac{dm}{dt} = \int_S D_v \nabla\rho_v \cdot \hat{n} dS = -D_v \frac{1}{\epsilon_0} C_e \left(\rho_{v,S} - \rho_{v,\infty}\right) \tag{2.46}$$

Note that if the capacitance is that of a spherical conductor ($C_e = 4\pi\epsilon_0 a$) we get the same result as for a drop. We make some changes to the definition of C_e, since there is no need to use ϵ_0 and we also multiply by 4π to get:

$$\frac{dm}{dt} = 4\pi C D_v \left(\rho_{v,\infty} - \rho_{v,S}\right) \tag{2.47}$$

Look similar to the case for a drop? So we can just take the result we derived previously:

$$\frac{dm}{dt} \cong 4\pi C \frac{s_i - 1}{\frac{R_v T_\infty}{e_{si}(T_\infty) D_v} + \frac{L_s}{T_\infty k}\left(\frac{L_s}{R_v T_\infty} - 1\right)} \tag{2.48}$$

Note the changes are that we now use C instead of a, s_i instead of s_l, e_{si} instead of e_{sl} and L_s (latent heat of sublimation) instead of L_v.

For a simple, thin hexagonal plate we calculate the capacitance of a circular disk or radius a, i.e. $C_0 = \frac{2a}{\pi}$. Nowadays computer models can calculate the capacitance of more complex geometries, but McDonald (1963) measured the electrostatic capacitance of brass ice crystal models and came up with shape factors to multiply C_0 by to get the capacitance of other geometries—see Figure 2.5. The shape factor, f, can be used to calculate the capacitance of other crystals by $C = f \times C_0$.

2.6 Vapour growth and nucleation within clouds

Now we have covered CCN, IN and growth by vapour diffusion of single drops and ice crystals lets start to put all of these things together—see Figure 2.6. Note that in general because IN are typically in much lower concentrations than CCN the number of ice crystals formed by heterogeneous ice nucleation is much much less than that formed by homogeneous ice nucleation. This can have an unexpected effect on the ice crystal number.

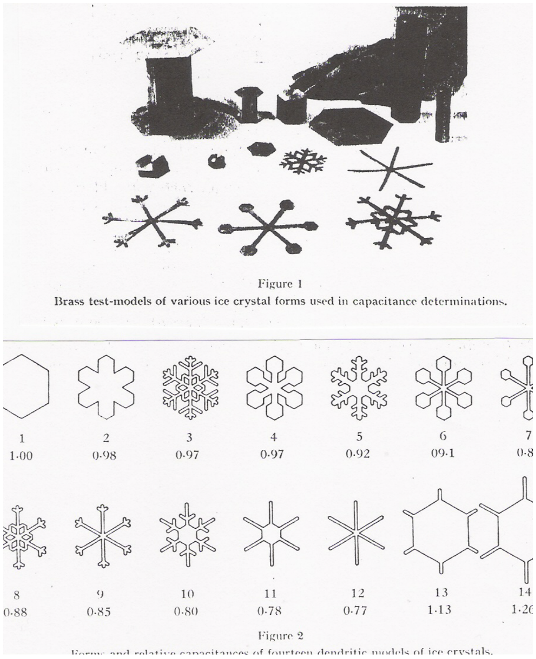

Figure 2.5: McDonald's early work showed that the electro-static analogy works very well for ice crystal growth.

Example 2.4 *Derive the 'radial' growth vs time of an ice crystal with circular disk morphology, with initial starting radius a_0.*

Other questions to consider:

- For the problem above what is the radial growth rate after 100 seconds? and therefore what is the temperature of the ice crystal? (assume the saturation ratio s_i = 1.10 and $T_\infty = -15$°C and the intial size is $a_0 = 5\mu$m).
- What is the radial growth rate of a drop under the same conditions?
- If there are 100 mg^{-1} of drops radius 10μ m at -36°C how long in seconds would it take to freeze halve of them by homogeneous freezing?

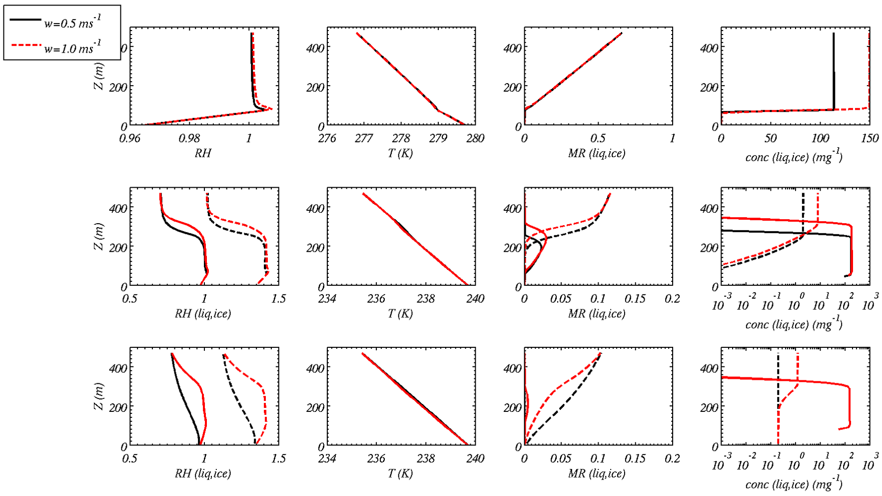

Figure 2.6: Top 4 figures shows a comparison between two model simulations of a warm cloud. It can be seen that the droplet number increases with increasing vertical wind because the supersaturation at cloud base increases; Middle 4 figures shows a comparison between two model simulations of a cold cloud where only homogeneous nucleation can act. It can be seen that liquid water condenses first (supercooled liquid water) and then homogeneous nucleation occurs. The final ice crystal number concentration is highest in the case with the highest updraft. Once ice crystals form they remove supersaturation and conditions approach ice saturation (equilibrium). The evaporation of drops in favour of ice crystal growth is known as the *Bergeron-Findesen Process*; Bottom 4 figures shows a comparison between two model simulations the same cold cloud as middle except that there are some heterogeneous ice nuclei in addition to homogeneous nucleation. In the case of the slow updraft conditions of water saturation are not reached because the low number of ice crystals is enough to deplete the supersaturation so that supercooled liquid water cannot form. In the case with the high updraft speed saturation over liquid water is reached and therefore both heterogeneous and homogeneous nucleation can occur. ∴ adding IN in a cold cloud tends to reduce the total crystal number.

THE KEY POINTS TO TAKE HOME HERE ARE:

- Be able to apply the equations of ice crystal growth for crystals of different morphology.
- Understand different ice nucleation modes and the variables important to ice nucleation in clouds.
- Be able to calculate the number of ice crystals formed by homogeneous nucleation.
- Understand how these processes interact within clouds.
- Know what the Bergeron–Findeisen (B–F) mechanism is and why it can lead to precipitation.