1.7 Particle size distributions

Until now we have only considered the bulk thermodynamics of clouds; real clouds contain particles. In some of the earlier problems you were asked what the size of the particles in the cloud were, given a total liquid water mass and a total number concentration of particles—you will have assumed they were the same size. In real clouds the particles are not all the same size but are distributed in size. You already know this because you know that sometimes clouds rain! The reason they rain is because some cloud particles grow preferentially in size and become rain drops: the drag force of drops falling in air is proportial to their projected area, while their weight is proportial to their volume, hence their terminal velocity increases with size. We need to become familiar with describing particle distributions this because, as we shall see, the rate that particles (drops / ice crystals) grow within a cloud depends strongly on their size.

Some reasons cloud particles become distributed in size are:

- Not all cloud drops / ice crystals form at the same time so they are not given the same growth time.
- Real clouds have small variations in the temperature and vapour field within them, meaning that some particles grow from the vapour quicker than others.
- Certain processes that occur in clouds happen in only localised regions of the cloud.

1.7.1 Measurements of cloud particle size distributions

There are several ways of measuring the size-distribution of particles within a cloud. The most common way is to use an instrument that consists of a continuous wave laser which falls onto a detector (or dump spot). When a particle passes through the beam the intensity of the signal is recorded and this can be converted to particle size. Therefore the number of particles breaking the beam per second and the corresponding size of the particle is noted. Gradually a size distribution is built up from these individual particle events.

Different instruments have different intervals between the sizes that they measure and so to make comparisons between the instruments fair we tend to divide by the interval width, hence we define a size distribution as $\frac{dN(D)}{dD}$, where $dN(D)$ is the number of particles between D and $D + dD$ and D is the diameter. This has units of m^{-4}.

1.7.2 Parameterisation of cloud particle size distributions

A useful way of representing size distributions is with the generalised gamma distribution:

$$\frac{dN(D)}{dD} = n_0 D^{\mu} \exp\left(-\lambda_0 D\right) \tag{1.29}$$

here n_0, μ and λ_0 are constant parameters depending on the distribution being parameterised. By comparison to a straight line n_0 (m^{-4}) is called the intercept and λ_0 (m^{-1}) is called the slope of the distribution.

It has been found that on many occasions exponential spectra (i.e. $\mu = 0$) do a reasonable job of explaining the size distribution of particles within clouds. This is useful because exponential spectra can easily be integrated analytically.

1.7.3 Moments of cloud particle size distributions

The moment of a distribution is the integral of the product of the distribution and the independent variable, in this case D. E.g. the zeroth moment, M_0, is just the total number concentration (m^{-3})

$$M_0 = \int_0^\infty \frac{dN(D)}{dD} dD \tag{1.30}$$

and the 2nd moment, M_2 is proportional to the total surface area of the drops per m^3 of air:

$$M_2 = \int_0^\infty \frac{dN(D)}{dD} \times D^2 dD \tag{1.31}$$

specifically, the total integrated area, $Area = \frac{\pi}{4} M_2$. The 3rd moment, M_3 is proportional to total mass of the water drops per m^3 of air:

$$M_3 = \int_0^\infty \frac{dN(D)}{dD} \times D^3 dD \tag{1.32}$$

specifically, the liquid water mixing ratio, $w_l = \frac{\pi\rho}{6} M_3$.

The integral of the generalized gamma distribution is defined as:

$$\int_0^\infty n_0 D^\mu \exp\left(-\lambda_0 D\right) dD = n_0 \frac{\mu!}{\lambda_0^{\mu+1}} \tag{1.33}$$

therefore if we know n_0 and λ_0 we can calculate any moment of the distribution. For example the zeroth moment:

$$M_0 = \int_0^\infty n_0 \exp\left(-\lambda_0 D\right) dD = n_0 \frac{0!}{\lambda_0^1} \tag{1.34}$$

and the 2nd moment:

$$M_2 = \int_0^\infty n_0 D^2 \exp\left(-\lambda_0 D\right) dD = n_0 \frac{2!}{\lambda_0^3} \tag{1.35}$$

and the 3rd moment:

$$M_3 = \int_0^\infty n_0 D^3 \exp\left(-\lambda_0 D\right) dD = n_0 \frac{3!}{\lambda_0^4} \tag{1.36}$$

and so on.

Calculation of the fifth moment is important in precipitation forecasting—why?

1.7.4 Relationship between the particle size distribution and albedo

The cloud albedo is the fraction of total incident radiation that is reflected / scattered back to space by the cloud. We will not cover the physics of scattering of light by cloud particles in this course. Instead we will just quote the result that an excellent approximation to the albedo, A_c, of a liquid water cloud can be calculated using:

$$A_c \cong \frac{\pi M_2 \Delta Z}{\pi M_2 \Delta Z + 15.4} \tag{1.37}$$

1.8 Latham and Salter's cloud brightening scheme

Cloud brightening `http://en.wikipedia.org/wiki/Cloud_reflectivity_modification` is one of the favoured options for mitigating the warming that occurs through a doubling of CO_2. It is a so called geoengineering scheme.

The idea here is to seed maritime stratocumulus clouds with small sea salt particles; doing so increases the cloud albedo, A, because it increases the number concentration of particles in the cloud. It does not increase the mass in the cloud, which is defined by bulk thermodynamics (Section 1.5).

So if in the case of no seeding we have the zeroth moment, $M_0 = n_0 \frac{1}{\lambda_0}$ and the third moment $M_3 = n_0 \frac{6}{\lambda_0^4}$ then taking the ratio of these we have $\lambda_0 = \left(6\frac{M_0}{M_3}\right)^{1/3}$ and thus $n_0 = M_0 \left(6\frac{M_0}{M_3}\right)^{1/3}$.

From this we can calculate that the second moment, M_2 is:

$$M_2 = M_0 \left(6\frac{M_0}{M_3}\right)^{1/3} \frac{2}{\left(6\frac{M_0}{M_3}\right)} \cong 0.6057 M_0^{1/3} M_3^{2/3} \tag{1.38}$$

checking this is dimensionally correct we see that the third moment ($\cong D^3$) is raised to the power 2/3 which leaves $\cong D^2$, which is what we want. From this we see that if we increase the number of cloud drops the second moment, M_2, of the distribution increases as $M_0^{(1/3)}$. Also from Equation 1.37 we see that increasing M_2 also increases the albedo A_c.

The change in albedo that can be achieved with this method is highest when the number of drops in the cloud is naturally low. If conditions are already polluted then the scheme loses its effectiveness.

Example 1.1 *A stratocumulus cloud, 100m thick, has a liquid water mixing ratio of* 0.5 *g m*$^{-3}$ *and a number concentration of* 50 *drops cm*$^{-3}$ *of air. Over a period of time you are able to increase the concentration of salt particles entering the cloud base to* 500 *drops cm*$^{-3}$ *of air. In both cases the size distribution can be considered to be exponentially distributed. What is the change in the cloud albedo?*

THE KEY POINTS TO TAKE HOME HERE ARE:

- Particle size distributions are parameterised using exponential spectra.
- Knowledge of two moments is enough to fit a moment conserving distribution.
- Any moment of the distribution can then be calculated.
- Albedo of stratocumulus clouds can be increased by addition of salt particles to the air entering the cloud.
- The susceptibility is highest for pristine clouds (i.e. clouds with low natural numbers of aerosol).
- You should be able to describe the principles involved in the cloud brightening scheme using the physics discussed in these lectures.
- You should be able to calculate the change in albedo resulting from adding sea water particles to the cloud.

1.9 More details on geoengineering clouds

It is thought that reflecting ~ 3.7 W m^{-2} of energy back to space will be enough to stop catastrophic warming over the next 50 years due to the rising levels of CO_2.

Question: how much sea-spray would we need to do this?

1.9.1 From Latham et al (2008, Phil Trans)

The average solar irradiance F (W m^{-2}) received at the Earth's surface is:

$$F = 0.25F_0\left(1 - A_p\right) \tag{1.39}$$

Where, F_0 is the solar flux at the top of the atmosphere and A_p is the planetary albedo. This can be derived by considering the solar flux at the Earth's surface, $F_0 = 1370$ W m^{-2}, falling on a disk with the radius of Earth and the average solar irradiance, F being over the whole planet.

A change in albedo, ΔA_p, produces a forcing ΔF of:

$$\Delta F \cong -340\Delta A_p \tag{1.40}$$

If $f_1 = 0.7$ is the fraction of Earth's surface covered by ocean, $f_2 = 0.25$ is the fraction of the ocean surface covered by marine stratocumulus clouds and f_3 is the fraction of those clouds that are seeded, then the average change in cloud albedo, ΔA_c required for a change in planetary albedo, ΔA_p is:

$$\Delta A_c = \Delta A_p/(f_1 f_2 f_3) \cong -\Delta F/60 f_3 \tag{1.41}$$

which if $f_3 = 1$ then we need to change the cloud albedo by 0.062 or the planetary albedo by 0.011.

I've shown how to calculate change in albedo previously, but a simpler relation can be shown to be

$$\Delta A_c \cong 0.075 \ln\left(N/N_0\right) \tag{1.42}$$

where N is the drop concentration in seeded clouds and N_0 the same but in unseeded clouds.

Equations 1.41 and 1.42 together can be written:

$$-\Delta F \cong 4.5 f_3 \ln\left(N/N_0\right) \tag{1.43}$$

or

$$\left(N/N_0\right) \cong \exp\left(-\Delta F/\left[4.5 f_3\right]\right) \tag{1.44}$$

which if $-\Delta F = 3.7$ W m-2 is about 2.3.

Aerosol concentrations stay relatively constant over certain regions because there is a balance between being injected into the atmosphere and being removed.

We can estimate how much we would need to spray into a boundary layer to increase the levels up to N with N_0 the natural concentration level. The number of droplets per second that would need to be sprayed into the atmosphere is:

$$\frac{dn_{spray}}{dt} = (N - N_0)A_E H f_1 f_2 f_3/\tau \quad (1.45)$$

$$= N_0(N/N_0 - 1)A_E H f_1 f_2 f_3/\tau \quad (1.46)$$

where A_E is the surface area of Earth $\sim 5{\times}10^{14}\text{m}^2$ and H is the depth of the boundary layer (typically 1000 m) and τ is the residence time of aerosols in the atmosphere $\sim$ 3 days or 2.6×10^5 seconds.

In order to estimate the volume of sea spray that must be introduced assume that the drops are all the same size and are about $a \cong 0.135\mu$m in radius; $N_0 \cong 100$ cm^{-3} or $\sim 10^8$ m^{-3}.

The volume spraying rate is therefore:

$$\frac{dV_{spray}}{dt} = \tfrac{4\pi}{3}\rho_w a^3 \times \tfrac{dn}{dt} \quad (1.47)$$

$$= \tfrac{4\pi}{3}\rho_w a^3 \times N_0(N/N_0 - 1)A_E H f_1 f_2 f_3/\tau \quad (1.48)$$

which is approximately 460 m^3 per second!!