A.4 Lecture 3

Some of the examples on CCN activation could in principle be used in the exam although you would be given Equations 2.21 and 2.22. The questions on droplet growth, or ice crystal growth can be solved analytically and so could be used in the exam. You would be given any of Equation 2.25, 2.27, 2.36 or 2.37 and be expected to be able to use them. The purpose of the spreadsheet is to show that the analytical solution is a very good approximation to the exact solution (which can only be solved iteratively).

Example 2.3 A CCN counter is used to measure the number of active CCN verses supersaturation at the ground. From analysis of the data a power-law (Equation 2.21) is fitted to the data, and the parameters of the fit are $C = 200$ cm^{-3} and $k = 0.50$. Calculate the number of cloud drops activated at cloud base for a wind speed of $w = 0.2, 0.5, 1.0$ and 5.0 ms^{-1}.

Answer Using Equation 2.22, $N_c \cong 0.88{\times}200^{2/(0.50+2)}\left[70 \times 0.2^{3/2}\right]^{0.50/(0.50+2)} \cong 88.03$mg^{-1}.

Example 2.4 How long does it take for a drop of diameter 10μm to grow to a precipitation-sized drop of diameter 2 mm given an in-cloud supersaturation of 2%? Assume the temperature is 290k, pressure is 900hPa and calculate the diffusivity and thermal conductivity from the expressions for D_v and k in your notes. Use Equation 1.11 for e_s.

Answer For this use Equation 2.37 and recognise that all terms on the right hand side are a constant,

- So that after integration the solution is $a(t) = \sqrt{2At + a_0^2}$, where $A \cong 2.3 \times 10^{-12}$ m^{2}s^{-1} is everything on the right hand side of Equation 2.37.
- Rearrange for t: $t = \frac{a(t)^2 - a_0^2}{2A}$.
- Substitute $a(t) = \frac{a(t)^2 - a_0^2}{2A} \cong \frac{\left(1\times10^{-3}\right)^2 - \left(5\times10^{-6}\right)^2}{2\times2.3\times10^{-12}} \cong 2.2 \times 10^5$ s or 2.5 days!

Example 2.4a For the problem above what is the drop growth rate after 100 s?

Answer Could do this a number of ways, try differentiating $a(t) = \sqrt{2At + a_0^2}$: $\frac{da}{dt} = \frac{A}{\left(2At+a_0^2\right)^{1/2}}$ and substitute $t = 100$s so $\frac{da}{dt} \cong 1 \times 10^{-7}$ ms^{-1} or 0.1μm s^{-1}. The size $a(t = 100s) \cong 21\mu$m.

Example 2.4b With reference to the previous questions on drop growth what is the temperature of the drop in the above question after 100s?

Answer This requires the balance of Fourier's law with the latent heat released by the mass growth rate by diffusion (Equation 2.29).

- The mass growth rate can be written in terms of the radial growth rate: $\frac{dm}{dt} = 4\pi a^2 \rho_w \frac{da}{dt}$.
- So using the numbers for a and $\frac{da}{dt}$ in the previous question we have $\frac{dm}{dt} \cong 4\pi \times \left(21 \times 10^{-6}\right)^2 \times 1000 \times 1 \times 10^{-7} \cong 6.3 \times 10^{-13}$kg s^{-1}.

- Multiply the mass growth rate by the latent heat of vapourisation and equate to Fourier's law: $6.3 \times 10^{-13} \times 2.5 \times 10^{6} \cong 4\pi \times ak\,(T_a - T_\infty)$.
- Rearranging for T_a: $T_a - T_\infty \cong 0.2295$. So the drop temperature is elevated above 290K by 0.2 K. Incidentally you do not need to include the time or size of the drop in this calculation as they cancel so at all times and all drops sizes the difference in drop and ambient temperature will be the same. Can you proove this?

Example 2.4c Formation of rain requires particles of different sizes, which have different terminal fall-speeds. Given this can you say anything about why the growth of drops by vapour diffusion is not able to explain the formation of rain in real clouds?

Answer Use $a(t) = \sqrt{2At + a_0^2}$. If we start with a small drop radius a_0 and a slightly larger drop with radius a_1 then the difference between the squares of these two drops with time is $2At + a_1^2 - \left(2At + a_0^2\right) = a_1^2 - a_0^2$. If the difference of the squares of the drops is constant with time this means that they get closer together in size. After an infinite amount of time the drop size distribution will be infinitesimally narrow and so if they were precipitation sized the whole cloud would fall as rain.