

# Physics of cloud formation

Now we have covered the necessary basic thermodynamics and definitions for cloud formation and also noted how particle sizes are distributed in clouds we will now start to consider the necessary physics of how cloud particles form and grow into the sizes observed in clouds. We will first cover how cloud drops are formed, then we will cover the subsequent growth of cloud drops and ice crystals.

## 2.1 Fick's law of diffusion and Fourier's law of heat transfer

These two laws can be used to great effect in describing the initial growth of water drops and ice crystals.

### 2.1.1 Diffusion of water vapour

If  $\rho_v$  represents the concentration of water vapour mass then the current density of water vapour is given by Fick's 1st law:

$$\bar{\mathbf{j}} = -D_v \nabla \rho_v \quad (2.1)$$

where  $D_v$  is the diffusivity of water vapour in air ( $\text{m}^2\text{s}^{-1}$ ) and  $\bar{\mathbf{j}}$  is the diffusion flux ( $\text{kg m}^{-2}\text{s}^{-1}$ ). This law describes the observation that vapour moves from high to low concentrations in order to smooth out gradients. Note that  $D_v$  depends on  $T$  and  $P$ :  $D_v = 2.11 \times 10^{-5} \left(\frac{T}{273.15}\right)^{1.94} \left(\frac{101325}{P}\right)$ .

Another thing to note is that the vapour mass is conserved, so similar to Maxwell's equations (for an incompressible flow) we have a continuity equation:

$$\frac{\partial \rho_v}{\partial t} = -\nabla \cdot \bar{\mathbf{j}} \quad (2.2)$$

this states that the local time rate of change of the vapour field is equal to the net outflow and inflow of vapour mass,  $\bar{\mathbf{j}}$  (current density).

Substitution of Eq 2.1 into Eq 2.2 yields Fick's second law of mass diffusion:

$$\frac{\partial \rho_v}{\partial t} = D_v \nabla^2 \rho_v \quad (2.3)$$

So the law governing diffusion of mass shows that the local time rate of change of mass in a volume is proportional to the second gradient of the mass field.

### 2.1.2 Diffusion of heat

Perhaps not unsurprisingly because the transfer of heat by conduction is governed by movement of particles (vibrations of molecules and electrons in a solid and collisions and diffusion of molecules during random motion in a gas) the law governing

heat transport by conduction is very similar to mass transfer by diffusion. It is called Fourier's law:

$$\bar{\mathbf{j}}_h = -k\nabla T \quad (2.4)$$

where  $k$  is the thermal conductivity of the air and  $\bar{\mathbf{j}}_h$  is the current density of heat. Note that  $k$  depends on  $T$ :  $k = (4.40 + 0.071T) \times 10^{-3}$

Assuming the air is relatively stationary then the local change of temperature of the air will depend on the current density of heat to the air. If this heat is added at a constant pressure (usually does) then the rate of change of temperature due to the addition of heat will be:

$$\frac{\partial T}{\partial t} = -\frac{\nabla \cdot \bar{\mathbf{j}}_h}{\rho c_p} \quad (2.5)$$

substituting Eq 2.4 in Eq 2.5 gives us:

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T \quad (2.6)$$

where  $\kappa = \frac{k}{\rho c_p}$ .

### 2.2.2 Cloud condensation nucleus counter

This is an instrument that measures the number of CCN in the atmosphere. It makes use of Fick's 2nd law and the result of Fourier's law between two flat parallel plates that have different temperatures. In this case there will be a continuous flow of heat and water vapour from one plate to the other so the set-up is usually called a continuous flow diffusion chamber.

- Have two parallel plates adjacent parallel plates held at different temperatures,  $T_1$  and  $T_2$ .
- Coat them both with water so that the vapour pressure on both plates is at water saturation for the temperature of the plate  $e_s(T_1)$  and  $e_s(T_2)$ .
- Flow filtered (letting CCN with sizes less than  $\cong 1\mu\text{m}$  through) air with CCN suspended in it through the parallel plates.
- The CCN in the air will be exposed to a supersaturation and will start to grow into cloud drops.
- Measure the size of the particles that come out of the instrument and count them with an optical particle detector if they are bigger than  $1\mu\text{m}$ .

Equations 2.3 and 2.6 can be applied for the steady state case, since neither change with time:

$$\nabla^2 \rho_v = 0 \quad (2.15)$$

$$\nabla^2 T = 0 \quad (2.16)$$

which have solutions:

$$\rho_v(x) = \rho_v(0) + \frac{\rho_v(L) - \rho_v(0)}{L}x \quad (2.17)$$

$$T(x) = T_1 + \frac{T_2 - T_1}{L}x \quad (2.18)$$

where  $L$  is the separation distance between the plates. Note that from the ideal gas law  $\rho_v = \frac{e}{R_v T}$  so that:

$$\frac{e(x)}{T(x)} = \frac{e_s(T_1)}{T_1} + \frac{e_s(T_2)/T_2 - e_s(T_1)/T_1}{L}x \quad (2.19)$$

and that the saturation vapour pressure between the plates:

$$e_s(x) = e_s(T(x)) \quad (2.20)$$

The CCN counter is usually operated for a range of temperature differences between the plates so that one can derive a number of ccn vs supersaturation plot—so called a CCN activity spectrum.

### 2.2.3 Measured concentrations of CCN

It is often found that measurements in the atmosphere using a CCN counter can be fitted to a power law:

$$N_c = CS^k \quad (2.21)$$

where  $N_c$  is the number concentration of activated CCN and  $S$  is the percent supersaturation. For polluted air  $C \cong 300 - 3000 \text{ cm}^{-3}$  and  $k = 0.2 - 2.0$  and for clean air  $C \cong 30 - 300 \text{ cm}^{-3}$  and  $k = 0.3 - 1.0$ .

It has been found that the number of cloud drops is a function of the vertical wind speed in the cloud (Twomey, 1959), because this determines the rate of cooling and therefore the peak supersaturation in the cloud. The reason for this is even though supersaturation may be reached it takes time for vapour molecules to diffuse to the CCN; hence a higher supersaturation can be achieved for a higher vertical wind:

$$N_c \cong 0.88C^{2/(k+2)} [70w^{3/2}]^{k/(k+2)} \quad (2.22)$$

where  $w$  is the vertical wind speed in  $\text{m s}^{-1}$ .

**Example 2.2** A CCN counter is used to measure the number of active CCN versus super-saturation at the ground. From analysis of the data a power-law (Equation 2.21) is fitted to the data, and the parameters of the fit are  $C = 200 \text{ cm}^{-3}$  and  $k = 0.50$ . Calculate the number of cloud drops activated at cloud base for a wind speed of  $w = 0.2, 0.5, 1.0$  and  $5.0 \text{ m s}^{-1}$ .

### 2.3 Growth of single cloud drops

In the case of diffusion of heat and mass between two parallel plates we assumed a steady-state so that the diffusivity and thermal conductivity could be neglected. For growth of droplets in the atmosphere we again assume a steady-state for Equation 2.3, but write down the Laplacian in radially symmetrical coordinates as we are considering diffusion to a sphere:

$$\nabla^2 \rho_v = \frac{d^2}{dr^2}(\rho_v) + \frac{2}{r} \frac{d}{dr}(\rho_v) = 0 \quad (2.23)$$

One can see by substitution that a solution to this is  $\rho_v(r) = \rho_{v,\infty} + (\rho_{v,a} - \rho_{v,\infty})\frac{a}{r}$ , where  $\rho_{v,\infty}$  is the vapour density at  $\infty$  and  $\rho_{v,a}$  is the vapour density on the boundary of the drop  $a$ . To get the growth rate of the drop, we need to compute the current density (or flux) of water vapour entering the sphere per unit time and integrate over the surface of the sphere. From Equation 2.1 (i.e. Fick's 1st law) the current density is:

$$\bar{\mathbf{j}}(r) = -D_v \frac{d}{dr} \rho_v = -D_v \frac{(\rho_{v,\infty} - \rho_{v,a})a}{r^2} \quad (2.24)$$

which is the vapour current density measured away from the sphere (along  $r$ ). On the sphere's surface this current density or flux is  $-D \frac{(\rho_{v,a} - \rho_{v,\infty})}{a}$ . Integrated over the whole surface of the sphere we therefore have:

$$\frac{dm}{dt} = 4\pi a D_v (\rho_{v,\infty} - \rho_{v,a}) \quad (2.25)$$

and using the ideal gas law  $\rho = e/(R_v \times T)$ :

$$\frac{dm}{dt} = \frac{4\pi a D_v}{R_v} \left( \frac{e_\infty}{T_\infty} - \frac{e_s(T_a)}{T_a} \right) \quad (2.26)$$

Using similar arguments for the transfer of heat it can be shown that:

$$\frac{dq}{dt} = 4\pi a k (T_\infty - T_a) \quad (2.27)$$

If we assume that upon condensation the latent heat release is removed by the heat flux away from the drop:

$$L_v \frac{dm}{dt} = -\frac{dq}{dt} \quad (2.28)$$

So substituting Equation 2.27 in Equation 2.28 we obtain:

$$\frac{dq}{dt} = -L_v \frac{dm}{dt} = 4\pi a k (T_\infty - T_a) \quad (2.29)$$

which can be rearranged to give

$$T_a = T_\infty + \frac{L_v}{4\pi a k} \frac{dm}{dt} \quad (2.30)$$

So heat is being added to the drop and being transferred away. The drop heats up because when the vapour molecules become incorporated into the drop they lose their internal energy which is transferred to latent heat of vapourisation. Equations 2.25 and 2.29 were first derived by Maxwell. They along with an equation for the saturation vapour pressure (Equation 1.11) give two equations with two unknowns and can be solved iteratively using a computer.

To form an analytic expression one can make use the Clausius-Clapyron equation:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \therefore \quad (2.31)$$

$$e_s(T_a) = e_s(T_\infty) \exp\left(\frac{L_v}{R_v} \left(\frac{T_a - T_\infty}{T_a T_\infty}\right)\right) \quad (2.32)$$

$$e_s(T_a) \cong e_s(T_\infty) \left(1 + \frac{L_v}{R_v} \left(\frac{T_a - T_\infty}{T_\infty^2}\right)\right) \quad (2.33)$$

and Equations 2.26 and 2.30 to give:

$$\frac{dm}{dt} \cong \frac{4\pi a D_v e_s(T_\infty)}{R_v T_\infty} \left(s_l - \frac{1 + \frac{L_v \delta}{R_v T_\infty}}{1 + \delta}\right) \quad (2.34)$$

where  $\delta = \frac{L_v}{4\pi a k T_\infty} \frac{dm}{dt}$  is very small and so, making approximations  $\frac{1}{1+\delta} = 1 - \delta$  and  $\frac{\delta}{1+\delta} = \delta$  the expression in the brackets becomes  $s_l - 1 + \delta \left(1 - \frac{L_v}{R_v T_\infty}\right)$  or  $s_l - 1 + \frac{L_v}{4\pi a k T_\infty} \frac{dm}{dt} \left(1 - \frac{L_v}{R_v T_\infty}\right)$ . We can rearrange and make  $\frac{dm}{dt}$  the subject to get the result:

$$\frac{dm}{dt} \left(1 + \frac{D_v e_s(T_\infty) L_v}{k R_v T_\infty^2} \left[\frac{L_v}{R_v T_\infty} - 1\right]\right) \cong \frac{4\pi a D_v e_s(T_\infty)}{R_v T_\infty} (s_l - 1) \quad (2.35)$$

$$\frac{dm}{dt} \cong 4\pi a \frac{s_l - 1}{\frac{R_v T_\infty}{e_s(T_\infty) D_v} + \frac{L_v}{T_\infty k} \left(\frac{L_v}{R_v T_\infty} - 1\right)} \quad (2.36)$$

Now since  $\frac{dm}{dt} = \frac{d}{dt} 4/3\pi a^3 \rho_w = 4\pi a^2 \rho_w \frac{da}{dt}$  we can write:

$$a \frac{da}{dt} \cong \frac{s_l - 1}{\frac{\rho_w R_v T_\infty}{e_s(T_\infty) D_v} + \frac{\rho_w L_v}{T_\infty k} \left(\frac{L_v}{R_v T_\infty} - 1\right)} \quad (2.37)$$

$$a \frac{da}{dt} \cong A \quad (2.38)$$

$$(2.39)$$

where  $A$  is a constant depending on temperature and supersaturation. For constant supersaturation and temperature etc the result of this is:

$$a(t) = \sqrt{2At + a_0^2} \quad (2.40)$$

where  $a_0$  is the initial radius of the drop. So after all that the increase in size of a drop with time is a parabola. This equation shows that if a cloud contains some large particles and some small particles the difference between the squares of the particle sizes remains constant with time  $a_{0,1}^2 - a_{0,2}^2$  so if they are growing they will become closer together in size. Therefore this growth process alone cannot explain the observed size distribution of drops within a cloud.  $\therefore$  other growth processes need to be considered.

Now we have covered this theory for drops we can use it to determine the growth rate of ice crystals; however, we need to take into account the deviation from spherical geometry.

## 2.4 Ice nuclei

There has been some suggestions in the literature that because the number of CCN activated depends on supersaturation over liquid water then the number of Ice Nuclei (IN) activated depends on supersaturation over ice. This has led to the development of ice nucleus counters based on this principle.

### 2.4.1 Ice nucleus counter

Due to the success of the parallel plate continuous flow diffusion chamber for measuring CCN, similar instruments have been made to measure IN. The differences are:

- The temperature of the plates are held below the melting point for ice.
- The plates are coated with ice and not liquid water.
- Supersaturation is reached wrt ice and not necessarily water.

We will talk about some issues and types of IN in a couple of lectures time.

**Example 2.3** *How long does it take for a drop of diameter  $10\mu\text{m}$  to grow to a precipitation-sized drop of  $2\text{ mm}$  given an in-cloud supersaturation of  $2\%$ ? Assume the temperature is  $290\text{K}$ , pressure is  $900\text{hPa}$  and calculate the diffusivity and thermal conductivity from the expressions for  $D_v$  and  $k$  in your notes. Use Equation 1.11 for  $e_s$ .*

Other questions to consider:

- For the problem above what is the drop growth rate after 100 seconds? and therefore what is the temperature of the drop? (hint: differentiate and substitute in Equation 2.29).
- Formation of rain requires particles of different sizes, which have different terminal fall-speeds. Given this can you say anything about why the growth of drops by vapour diffusion is not able to explain the formation of rain in real clouds?

THE KEY POINTS TO TAKE HOME HERE ARE:

- CCN spectra are parameterised using a power-law.
- Fick's and Fourier's laws—can be used to solve for diffusion of mass and heat to / from drops.
- Explain why the temperature of a growing drop is warmer than it's surroundings.
- The final result that the growth rate is a parabola.
- Be able to calculate  $a(t)$  due to vapour diffusion.
- Understand how continuous flow diffusion chambers work and that CCN and IN counters operate on very similar principles.