## 2.2 Cloud condensation nuclei

One thing to note is that in the atmosphere practically all water drops within a cloud are formed on aerosol particles known as Cloud Condensation Nuclei (CCN). In a rising parcel, which therefore cools and saturates water will only condense if there is something for it to condense onto. CCN are the sites that water condenses onto; however, condensation does not occur *exactly* at the point of saturation, since CCN do not take up water straight away. A slight super-saturation is required for the CCN to become active condensation sites. The higher the super-saturation the more of the CCN become active and grow into cloud drops. Why?

### 2.2.1 aside Kelvin's equation—derivation not needed

The concept of surface tension, $\sigma$, is that it arises due to the force of cohesion between molecules in a liquid. The surface tension is the force exerted parallel to the surface divided by the length over which it acts: $\sigma = \frac{F}{L}$.

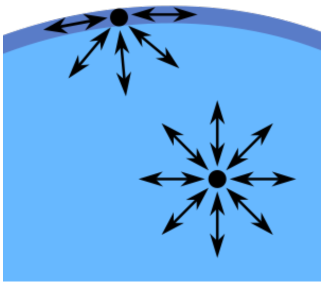

Figure 2.1: Taken from Wikipedia. Molecules inside the body of liquid experience forces pulling them in all direction (hence no net force). Molecules on the surface only experience the cohesive forces on one side and so are pulled inwards, eventually resulting in internal pressure (and force balance). If a molecule is moved away the lateral forces acting will tend to pull the molecule back into the bulk of the liquid.

Consider the 'thought experiment' in Figure 2.2, it demonstrates *capillary action* of water. If you partly submerse a non-wetting tube in water the water in the tube will form a spherical cap and will sink below the water line outside of the tube[1]. The reason is because the vapour pressure over the drop increases. We can calculate how much it increases by recognising that the pressure at depth $h$ is the same both inside and outside of the capillary. The increase in vapour pressure is provided by the extra weight of vapour on top of the spherical cap, but the remaining pressure increase is due to the surface tension (squeezing the molecules inside the drop so they repel each other). The pressure increase due to the surface tension is a function of the drop radius, so the drop will sink until the weight of air above

[1]If the tube is wettable you will get a concave 'cap' in the tube and the water level will rise.

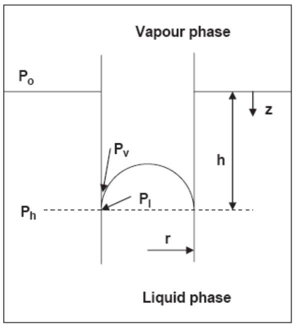


Figure 2.2: Taken from Galvin (2005). A non-wetting tube is inserted into the liquid, which causes a spherical cap to form. The vapour pressure far from the drop is $e_0$ and the pressure at $h$ is the weight of water plus vapour above it.

it, plus the pressure increase due to surface tension, equal the weight of water in a column outside of the tube. We will now derive it.

The total pressure due to the liquid and vapour at *h outside the tube* is:

$$e_h = e_0 + \rho_l g h \tag{2.7}$$

where $\rho_l$ is the density of water.

The total pressure at depth *h inside the tube* can also be written

$$e_h = e_0 + (e_v - e_0) + \Delta P_c \tag{2.8}$$

where $e_v$ is the vapour pressure at the drops surface and $\Delta P_c$ is the pressure difference between the outside and the inside of the drop. Note that the pressure must increase due to the fact that the drop is curved and has surface tension (i.e. molecules pulling inward, creating an internal pressure, and hence force that must be felt on a surface at $h$).

The vapour pressure is the weight of vapour above the surface of the drop hemisphere, which is described by the hydrostatic relation: $\frac{de}{dz} = \frac{eg}{R_v T}$. This can be integrated from 0 to the surface of the drop:

$$e_v = e_0 \exp\left(\frac{gh}{R_v T}\right) \tag{2.9}$$

By equating Equations 2.7 and 2.8 we can derive an equation for the depth that the water sinks to:

$$h = \frac{\cancel{e_v} + \Delta P_c - \cancel{e_0}}{\rho_l g} \tag{2.10}$$

since for all practical applications $\Delta P_c >> e_v - e_0$

Since forces are in balance, the pressure difference between drop and air must balance the surface tension.

The force due to the difference in pressure is $\Delta Pc \times r^2 \sin\theta d\theta d\phi$ ($\theta$ measured between $\hat{k}$, the unit vector along the $z$ axis) directed radially outwards. However, it is the component directed upward we are interested in as the lateral components cancel out. Hence, multiply by $\cos\theta$ and integrate over the hemisphere:

$$F = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} \Delta P_c r^2 \sin\theta\cos\theta d\theta d\phi \quad (2.11)$$

$$= \Delta P_c \pi r^2 \quad (2.12)$$

which must be balanced by the surface tension, $F_\sigma$, which is directed along the circumference of the circle in contact with the surface at $h$. Thus the surface tension force that balances the pressure force can be calculated as $F_\sigma = 2\pi r\sigma$. Thus we have:

$$\Delta P_c = \frac{2\sigma}{r} \quad (2.13)$$

Sub Equations 2.9 and 2.13 into 2.10 to get Kelvin's equation:

$$e_v = e_{s,drop} = e_0 \exp\left(\frac{2\sigma}{R_v T \rho_l r}\right) \quad (2.14)$$

This says that the smaller the water drop the higher the vapour pressure on the surface of the drop is. Thus *supersaturation* is required to grow small drops into larger drops.

| Droplet radius (nm) | 1000 | 100 | 10 | 1 |
|---|---|---|---|---|
| $e_v/e_0$ | 1.001 | 1.011 | 1.114 | 2.95 |

THE KEY POINTS TO TAKE HOME HERE ARE:

- Supersaturation is required to move water molecules to a curve surface.
- The higher the curvature the higher the supersaturation required.