Appendix A
Problems and solutions

A.1 Lecture 1 and 2

These examples on adiabatic liquid water mixing ratio and finding cloud base temperature and pressure are not on the exam. They are designed to help you become familiar with common terms in cloud physics and what they mean. You can also use the webpage to help you with the calculation if you like. Obviously you can't use a webpage in the exam but as I am sure you are aware as third / fourth year physicists that many problems in Physics do not have readily available analytical solutions. This is more so in research.

Example 1.1 A stratocumulus cloud has $T = 280$K and $P = 900$ hPa at cloud base. Cloud top is at a pressure of 870 hPa. What is the adiabatic liquid water mixing ratio?

Answer To answer this one you can either use the webpage to solve Equation 1.16 iteratively or use the approx. method in the notes. Here we use the approx. method.

- Use Equation 1.19 to calculate the conserved quantity, $\theta_{q,sat}$.
- $\theta_{q,sat}$ requires the saturation vapour pressure, which from Equation 1.23 is $610.7\exp\left(\frac{2.5\times10^6}{461}\left[\frac{T-273.15}{273.15^2}\right]\right) \cong 1004.7$ Pa.
- $\theta_{q,sat} \cong T\left(\frac{1000}{P}\right)^{R/c_p}\left(1+\frac{\epsilon e_s(T)L_v}{PTc_p}\right) \cong 280\left(\frac{1000}{900}\right)^{287/1005}\left(1+\frac{0.622\times1004.7\times2.5\times10^6}{90000\times280\times1005}\right) \cong$ 306.35 K.
- Now we must solve for T_2 in Equation 1.28. If all terms are evaluated we get $\Pi_2 = \left(\frac{1000}{870}\right)^{287/1005} \cong 1.041$ and $T_2 = \frac{\frac{1.7815\times10^7}{87000}+\frac{306.35}{1.041}}{1+\frac{6.868\times10^4}{87000}} \cong 278.98$ K.
- Now use the cloud base and cloud top T and P in Equation 1.18 to find the water content.
- $ALMR = 0.622\times610.7\times\left(\frac{\exp\left(\frac{2.5\times10^6}{461}\left[\frac{280-273.15}{273.15^2}\right]\right)}{90000}-\frac{\exp\left(\frac{2.5\times10^6}{461}\left[\frac{278.98-273.15}{273.15^2}\right]\right)}{87000}\right) \cong$ 0.27×10^{-3} kg/kg.

Example 1.1a What percentage of the total water mixing ratio is the adiabatic liquid water mixing ratio?

Answer The total water mixing ratio is the vapour content at cloud base. Why? because at cloud base there is no liquid water, only vapour. The vapour content at cloud base is the saturation vapour mixing ratio. Why? because at cloud base we are at water saturation. Any cooler and we have condensation.

- Use Equations 1.13 and 1.23 to find the saturation vapour mixing ratio:
- $e_{sw} = 610.7\exp\left(\frac{2.5\times10^6}{461}\left[\frac{280-273.15}{273.15^2}\right]\right) \cong 1004.7$ Pa.

- $w_s(T,P) = \frac{\epsilon e_s(T)}{P} = \frac{0.622\times1004.7}{90000} \cong 6.9\times10^{-3}$ kg/kg.
- So the fraction of liquid water to total water is the answer from Example 1.1 divided by 6.5×10^{-3}, or $0.27/6.9 \cong 0.04$.

Example 1.1b If the number concentration of particles is 200 mg^{-1} of air what is the mean diameter of the drops at cloud top?

Answer We know the total mass of drops in 1 kg of air (0.27×10^{-3}) and the number of drops per kg of air is 200×10^6 so if we assume they are all the same size we can calculate their mean diameter.

- The mass of one drop is total mass divided by total number: $\frac{0.27\times10^{-3}}{200\times10^6} \cong 1.35\times10^{-12}$ kg.
- Equate this to the mass of a sphere, density 1000 kg m^{-3}: $1.35\times10^{-12} = \frac{\pi}{6}\rho_w D^3$.
- Rearrange and solve for D: $D = \left(\frac{1.35\times10^{-12}\times6}{\pi1000}\right)^{1/3} \cong 1.4\times10^{-5}$m or 14 μm.

Example 1.1c If the liquid water mixing ratio is sub-adiabatic, say 70% of the adiabatic value what is the mean diameter of the drops assuming a drop concentration of 100 mg^{-1} of air?

Answer Very similar arguments to above.

- The mass of one drop is total mass divided by total number: $\frac{0.27\times10^{-3}\times70\%}{100\times10^6} \cong 1.89\times10^{-12}$ kg.
- After equating to the mass of a sphere, rearrange and solve for D: $D = \left(\frac{1.89\times10^{-12}\times6}{\pi1000}\right)^{1/3} \cong 1.53\times10^{-4}$m or 15 μm.

Example 1.1d If the same initial temperature and pressure are used, but the cloud top is 200m above cloud base what is the adiabatic liquid water mixing ratio? Use a temperature lapse rate of 6.5 K km^{-1}.

Answer This is an approximation because the lapse rate isn't exactly 6.5 K km^{-1}.

- The cloud top temperature is the cloud base temperature minus the cloud thickness multiplied by the lapse rate: $280 - 200\times6.5\times10^{-3} \cong 278.7$ K.
- The adiabatic liquid water mixing ratio (Equation 1.18) requires the cloud base and cloud top temperature and pressure. We have everything except cloud top pressure, so use the hydro-static equation to find cloud top pressure: $\frac{dP}{dz} = -\rho g$, or substituting the ideal gas law $\frac{dP}{dz} = -\frac{P}{R_a}Tg$.
- Solve the hydro-static equation by integrating both sides wrt z: $\int_{P0}^{P1}\frac{1}{P}dP = -\frac{g}{R_a}\int_{z0}^{z0+200}\frac{1}{T}dz$, but we know that $T(z) = 280 - z\times6.5\times10^{-3}$.
- Solving this yields $\ln\left(\frac{P1}{P0}\right) = \frac{g}{R_a\times6.5\times10^{-3}}\ln\left(\frac{280-200\times6.5\times10^{-3}}{280}\right)$
- Or solving for $P1$: $P1 = P0\exp\left(\frac{g}{R_a\times6.5\times10^{-3}}\ln\left(\frac{280-200\times6.5\times10^{-3}}{280}\right)\right)$.
- $P1 = 900\exp\left(\frac{9.8}{287\times6.5\times10^{-3}}\ln\left(\frac{280-200\times6.5\times10^{-3}}{280}\right)\right) \cong 878.3$ hPa.

- Now use the cloud base and cloud top T and P in Equation 1.18 to find the water content.
- $ALMR = 0.622 \times 610.7 \times \left(\frac{\exp\left(\frac{2.5\times10^6}{461} \left[\frac{280-273.15}{273.15^2} \right] \right)}{90000} - \frac{\exp\left(\frac{2.5\times10^6}{461} \left[\frac{278.7-273.15}{273.15^2} \right] \right)}{87830} \right) \cong$ 0.47×10^{-3} kg/kg.