Lecture 9 : Particle hydrodynamics

1st point : cloud drops fall with

$$C_D = \frac{24}{Re}$$

Rain drops. fall with

$$C_D \sim 0.5$$

C_D = drag coefficient

R = Reynolds number

$= \frac{u D \rho_a}{\eta}$

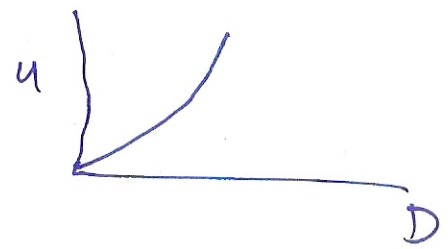
η = dynamic viscosity $[1.7 \times 10^{-5}\ \mathrm{Pa\,s}]$

Cloud drops :-

$$\frac{24\eta}{u D \rho_a} \times \frac{1}{2}\rho_a u^2 \frac{\pi D^2}{4} = \frac{\pi D^3}{6}(\rho_w - \rho_a) g$$

$$\boxed{u = \frac{1}{18}\frac{(\rho_w - \rho_a)}{\eta} g D^2}$$

$u \propto D^2$

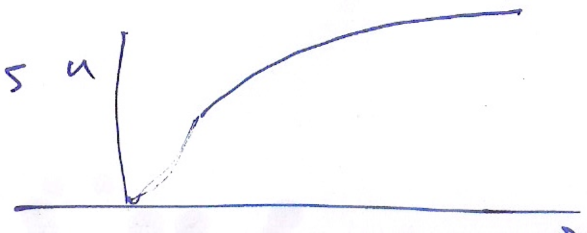


Rain drop:

$$0.5 \times \frac{1}{2}\rho_a u^2 \frac{\pi D^2}{4} = \frac{\pi D^3}{6}(\rho_w - \rho_a) g$$

$$\boxed{u = \sqrt{\frac{8}{3}\frac{(\rho_w - \rho_a)}{\rho_a} g D}}$$

$u \propto D^{0.5}$



Example : What is the u_t of 5mm diameter raindrop?

$$u = \sqrt{\frac{8}{3}\frac{(1000-1.2)\times 9.8\times 5\times 10^{-3}}{1.2}}$$

$$= 10.43 \ \mathrm{ms^{-1}}$$

Somehow a drop grows larger than neighbours

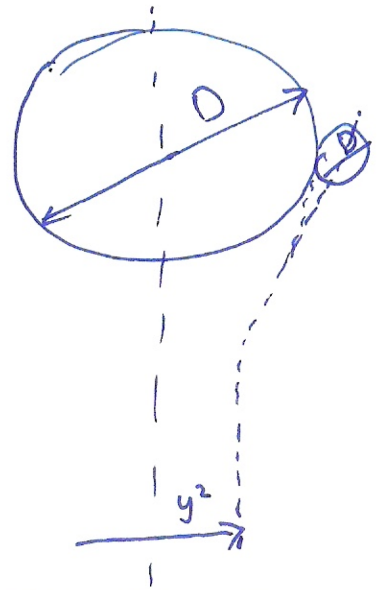
Define volume of air swept out s^{-1} in frame of drop.

$$V = \frac{\pi (D+d)^2}{4} \left| u(D) - u(d) \right| \qquad [m^3 s^{-1}]$$

Number conc, N $[m^{-3}]$

$V \times N$ $[\# s^{-1}]$

Define collision efficiency, $E(D,d)$



$$E(D,d) = \frac{y^2}{(D+d)^2}$$

the ratio of area that results in a grazing collision to the geometric sweep out area.

$\therefore$

$$\frac{dV}{dt} = \text{eq}^{\underline{n}}\ 4.8$$

$$\frac{dD}{dt} = \frac{E\, W_e\, a\, D^b}{2\rho_w}$$

$$D(t) = \left(\frac{E W_e a (1-b)}{2\rho_w} t + D_0^{1-b} \right)^{\frac{1}{1-b}}$$

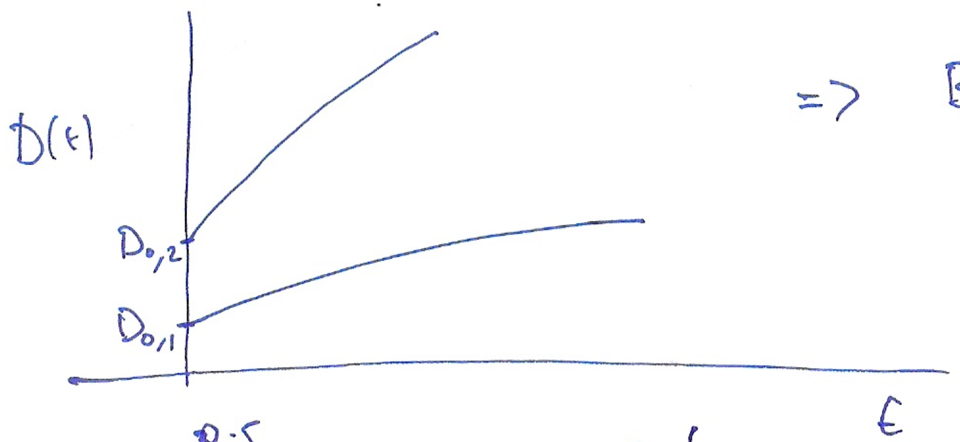
E = collision eff.

W_e = liquid water content [kg m

ρ_w = density of water [kg m

$a = \sqrt{\frac{8}{3} \frac{(\rho_w - \rho_a) g}{\rho_a}} \sim 1.47 \times 10^2$

$b = 0.5$



$\Rightarrow$ Broadens with time.

$$D^{0.5} = A + D_0^{0.5}$$

$D_0 = 9$, $A = 1$

$D = 16$

$\Delta D_0 = 7$

$\Delta D = 9$

$D_0 = 16$, $A = 1$

$D = 25$

Predicts cloud drop into raindrop in ~ 10 mins (c.f. days)

Eg.

$$\frac{4\rho_w}{E W_c a}\left(D^{0.5} - D_0^{0.5}\right) = t \quad , \text{ for } D_0 = 1\times10^{-4}, \; D = 5\times10^{-3}, \; W_c = 1\,g\,m^{-3}$$

$$t = 600\,s$$