Lecture 8.

Rate of change of supersaturation

$$\frac{dS_e}{dt} = (S_e+1)\left[\underbrace{\left(\frac{L_v}{c_p R_v T} - \frac{1}{R_a}\right)\frac{g w}{T}}_{\substack{\text{Adiabatic cooling}\\ \text{\& pressure reduction}}} - \underbrace{\left(\frac{1}{w_v} + \frac{L_v^2}{R_v T^2 c_p}\right)\frac{dw_e}{dt}}_{\substack{\text{reduction in } w_v\\ \text{due to condensation}\\ +\text{ increase in } T\\ \text{due to condensation}}} - \underbrace{\left(\frac{1}{w_v} + \frac{L_v L_s}{R_v T^2 c_p}\right)\frac{dw_i}{dt}}_{\substack{\text{reduction in } w_v\\ \text{due to sublimation}\\ +\text{ increase in } T \text{ due}\\ \text{to ice growth.}}}\right]$$

We can't solve analytically, but . . .

set $\frac{dS_e}{dt}$ to 0 [steady state]

We know previously that a single drop, mass m, radius a, grows from vapor:-

$$\frac{dm}{dt} = \frac{4\pi a S_l}{A_1} \qquad [\mathrm{kg\,s^{-1}}]$$

and a single ice Xtal, mass m, capacitance C:

$$\frac{dm}{dt} = \frac{4\pi C S_i}{A_2} \qquad [\mathrm{kg\,s^{-1}}]$$

Note that:

$$S_i = \frac{e}{e_{sat,i}} - 1$$

$$S_l = \frac{e}{e_{sat,l}} - 1$$

[e = vapour pressure, Pa
$e_{sat,l}$ = sat. vap. pressure w.r.t liquid, Pa
$e_{sat,i}$ = sat. vap. pressure w.r.t ice, Pa

$$\therefore \quad S_i = (S_l + 1)\frac{e_{sat,l}}{e_{sat,i}} - 1$$

[Supersat w.r.t ice in terms of Supersat. w.r.t liquid]

In order to get the rate of change of mass mixing ratio, $\dot{w}_l$ or $\dot{w}_i$ multiply these growth rates by the number density of drops or Xtals.

$$\therefore \quad \frac{dw_l}{dt} = N_{drops} \times \frac{4\pi\bar{a}\, S_e}{A_1} \qquad [\mathrm{kg\ kg^{-1}\ s^{-1}}]$$

$$\frac{dw_i}{dt} = N_{ice} \times \frac{4\pi\bar{c}}{A_2}\left[\frac{(S_e+1)e_{sat,l}}{e_{sat,i}} - 1\right]$$

S-S:

$$0 = \alpha w - \beta N_{drop}\,\bar{a}\, S_e - \gamma N_{ice}\,\bar{c}\,(S_e + \delta)$$

Rearrange for S_e :-

$$\boxed{S_e = \frac{\alpha w - \gamma N_{ice}\,\bar{c}\,\delta}{(\beta N_{drop}\,\bar{a} + \gamma N_{ice}\,\bar{c})}} \qquad [\text{Quasi-steady supersat.}]$$

Example

1. $T = 258\,K$, $P = 500\,hPa = 500 \times 10^2\,Pa$

Cloud with $w = 1\,ms^{-1}$, $N_{drops} = 100 \times 10^6\,kg^{-1}$, $\bar{a} = 30 \times 10^{-6}\,m$

$$\alpha = 6.6 \times 10^{-4}\ m^{-1}$$

$$\beta = 3.1 \times 10^{-4}\ kg\,m^{-1}s^{-1}$$

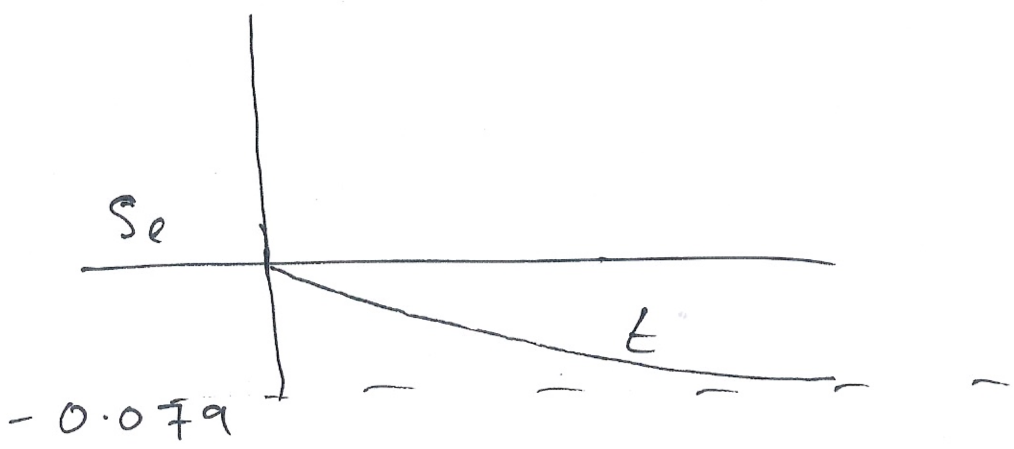
$$\gamma = 3.1 \times 10^{-4}\ kg\,m^{-1}s^{-1}$$

$$\delta = 0.1375$$

a) Steady supersaturation? $S_e = \dfrac{\alpha w}{\beta N_{drop}\bar{a}} = 7 \times 10^{-4}$

b) $N_{ice} = 10^6\,kg^{-1}$, $\bar{c} = 200 \times 10^{-6}\,m$

$$S_e = \frac{\alpha w - \gamma N_{ice}\bar{c}\delta}{\beta N_{drop}\bar{a} + \gamma N_{ice}\bar{c}} = -0.079$$ [cloud glaciates]



c) Increase $w = 25\,ms^{-1}$

$S_l = 0.008$ [cloud does not glaciate]

d) $w = 0\,ms^{-1}$, $N_{drop} = 0$

$S_l = -\delta$
$= -0.1375$ [this is ice saturation].