Lecture 7 (what factors affect S_l in clouds?).

We know :-

$$S_l = \frac{e - e_{sat,l}}{e_{sat,l}}$$

$S_l \equiv$ supersaturation

$e \equiv$ water vapour pressure, Pa

$e_{sat,l} \equiv$ sat. vap pres, Pa, wrt to liquid wat

diff. wrt time :

$$\frac{dS_l}{dt} = \frac{1}{e_{sat,l}}\frac{de}{dt} - \frac{e}{e_{sat,l}^2}\frac{de_{sat,l}}{dt} \quad \text{—} ①$$

We also know :-

*mistake in notes

$$w_v = \frac{\varepsilon e}{P}$$

$w_v \equiv$ mixing ratio of w/v [kg kg^{-1}]

$\varepsilon \equiv R_a/R_v = 0.622$

$P \equiv$ total pressure; Pa

$$\Rightarrow \quad e = \frac{w_v P}{\varepsilon}$$

diff. wrt time:

$$\frac{de}{dt} = \frac{1}{\varepsilon} P \frac{dw_v}{dt} + \frac{1}{\varepsilon} w_v \frac{dP}{dt} \quad \text{—} ②$$

Chain rule:-

$$\frac{de_{sat,l}}{dt} = \frac{de_{sat,l}}{dT}\frac{dT}{dt}$$

$$= \frac{L_v e_{sat}}{R_v T^2}\frac{dT}{dt} \quad — ③$$

$$\left[\frac{de_{sat,l}}{dt} = \frac{L_v e_{sat}}{R_v T^2} \right]$$ (Clausius-Clapeyron)

[$T \equiv$ temperature, K

L_v = Latent heat of vaporisation, $J\,kg^{-1}$]

We also know (1st law TD):-

$$\underline{c_p \frac{dT}{dt}} - R_a \frac{T}{P}\frac{dP}{dt} - L_v \frac{dw_l}{dt} - L_s \frac{dw_i}{dt} = 0 \quad — ④$$

[$c_p \equiv$ specific heat cap air 1005 $J\,kg^{-1}K^{-1}$

$L_s \equiv$ latent heat of sublimation, $J\,kg^{-1}$

w_l = liquid water mixing ratio

$w_i \equiv$ ice " " "]

Sub ④ in ③ & result + ② in ①

Using conservation of total water:

$$\frac{dw_v}{dt} + \frac{dw_l}{dt} + \frac{dw_i}{dt} = 0$$

& hydrostatic relation:

$$\frac{dP}{dz} = -\frac{P}{R_a T} g$$

$$\Rightarrow \frac{dP}{dt} = \frac{dP}{dz}\frac{dz}{dt} = \frac{-P}{R_a T} g\, w$$

[z = vertical coordinate

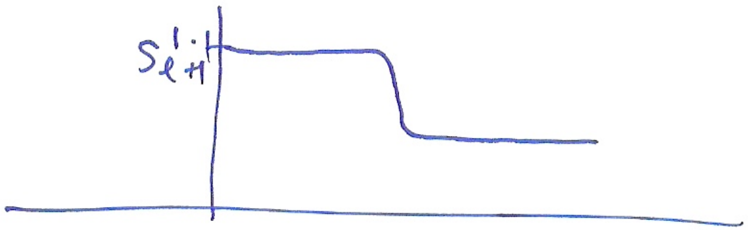
$w = \frac{dz}{dt}$, ms^{-1}]

* Slide *

Stable cloud (mixed phase) - freezer.

$$\frac{dS_l}{dt} \propto -a\frac{dw_l}{dt} - b\frac{dw_i}{dt}$$

why does S_l+1 stay at 1 for a time?



→ reservoir of water

$$\frac{dw_l}{dt} \propto N_d\,\bar{a}\,S_l$$

Why does S_l+1 stay at constant value after drops evaporate?

$$\frac{dw_i}{dt} \propto N_i\,\bar{C}\,S_i$$

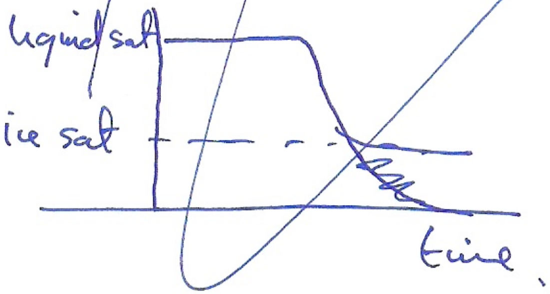
$$S_i = \left[(S_l+1)\frac{e_{sat,l}}{e_{sat,i}} - 1\right]$$

ice saturation.

$$\frac{dS_e}{dt} = (S_e+1)\left[\underbrace{\left(\frac{gL_v}{c_pR_vT^2} - \frac{g}{R_aT}\right)w}_{\text{cooling \& Pressure reduction}} - \underbrace{\left(\frac{1}{w_v} + \frac{L_v^2}{R_vT^2c_p}\right)\frac{dw_l}{dt}}_{\text{reduction in } w_v \text{ due to condensation + increase in } T \text{ due to condensation}} - \underbrace{\left(\frac{1}{w_v} + \frac{L_vL_s}{R_vT^2c_p}\right)\frac{dw_i}{dt}}_{\text{reduction in } w_v \text{ due to \sout{condensation} sublimation + increase in } T \text{ due to ice deposition.}}\right]$$

Lab experiment:

$$\frac{dS_e}{dt} = (S_e+1)\left[\,0 \ldots - a\frac{dw_l}{dt} - b\frac{dw_i}{dt}\right]$$



Ice cloud with vertical wind.

Why does S_i for $N_{ice} = 0.1\,mg^{-1}$ increase then decrease

$$\frac{dS_i}{dt} \propto a w - \frac{dw_i}{dt}$$

~~for~~ initially $\frac{dw_i}{dt}$ small, when Xtals grow becomes larger.

Why does S_i decrease fast when N_i high?

$$\frac{dw_i}{dt} \propto \enclose{circle}{N_i}\, \bar{C} \,\&\, S_i$$

Mixed phase.

~~[illegible]~~ ~~[illegible]~~ low N_i or large ω results in $S_i = 0$
ie/ liquid cloud possible.