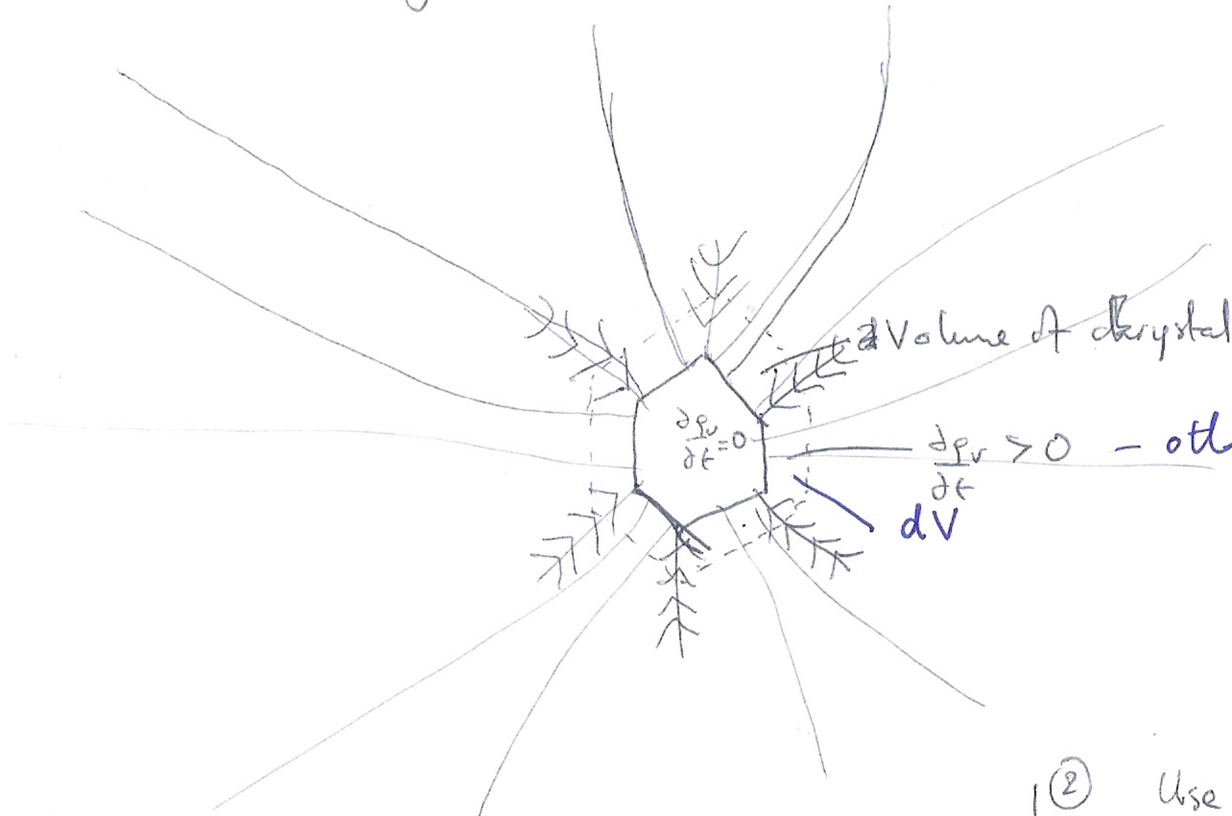
lecture 6

1) Consider ice crystal, mass m, in a vapour density field, $\rho_v(x,y,z)$



ρ_v = vapour density [$kg\,m^{-3}$]

$\underline{j}_m$ = current density of water vapour [$kg\,m^{-2}s^{-1}$]

① Continuity of vapour:-

$$\frac{\partial \rho_v}{\partial t} = -\nabla\cdot\underline{j}_m$$

The mass growth rate:-

$$\int \frac{\partial \rho_v}{\partial t}\,dV = -\int \nabla\cdot\underline{j}_m\,dV$$

up to boundary of Xtal

$$= \frac{dm}{dt}\text{ [of crystal]}$$

② Use Gauss' divergence theorem:-

$$\int \nabla\cdot\underline{j}\,dV = \int_S \underline{j}\cdot d\underline{S}$$

$$\therefore \quad \frac{dm}{dt} = -\int_S \underline{j}_m\cdot d\underline{S}$$

Recap:

$\underline{j}_m = -\nabla\rho_v$ [Fick's law]

$\underline{E} = -\nabla V$ [Electric field, scalar pot.]

2/ $\int_S \underline{E} \cdot d\underline{S} = \frac{Q}{\epsilon_0} = \frac{C_e(V_S - V_\infty)}{\epsilon_0}$ ✓

$Q = C_e(V_S - V_\infty)$

[C_e = electrostatic cap.
V_∞ = potential @ ∞
V_s = potential on surface.]

[Potential of capacitor ≡ charge / Capacitance]

By analogy :-

$-\int_S \hat{\underline{J}} \cdot d\underline{S} = \frac{C_e}{\epsilon_0}(\rho_\infty - \rho_s)$

↑ field lines come in

$\frac{dm}{dt} = 4\pi C(\rho_\infty - \rho_s)$

[Call $C_e = 4\pi\epsilon_0 C$ as we don't need ϵ_0]

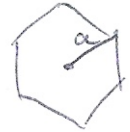
Recall we found for a drop:

$\frac{dm}{dt} = 4\pi a(\rho_\infty - \rho_s)$

So we can write (for crystal):

$\frac{dm}{dt} = \frac{4\pi C(s_i - 1)}{B(T,P)}$.

[C for hexagonal plate $\approx \frac{2a}{\pi}$



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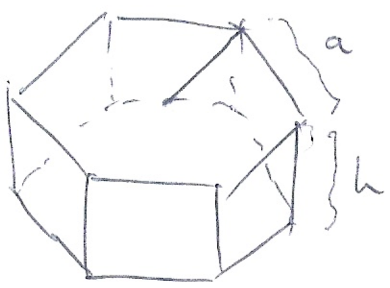
3/ Consider hex prism:



$$V = \frac{3\sqrt{3}}{2} a^2 h$$

$$m = \frac{3\sqrt{3}}{2} a^2 h \rho_i$$

$$\left[\rho_i = 920 \text{ kg m}^{-3}\right]$$

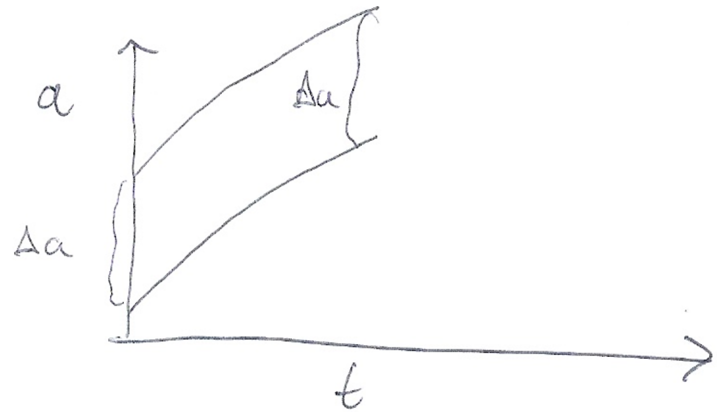
Change of variable: $\frac{dm}{dt} = \frac{3\sqrt{3}}{\cancel{2}} \times \cancel{2} a h \rho_i \frac{da}{dt}$

$$\therefore \quad 3\sqrt{3}\, \cancel{a} h \rho_i \frac{da}{dt} = \frac{\cancel{4\pi} \times 2\frac{\cancel{a}}{\cancel{\pi}} S_i}{\beta(T,P)}$$

$$h \cancel{a} \frac{da}{dt} = \frac{8}{3\sqrt{3}} \times \frac{S_i}{\rho_i \beta(T,P)}$$

Sol^n

$$\boxed{a(t) = a_0 + \frac{8}{3h\sqrt{3}} (\rho_i \beta)^{-1} S_i t}$$



4) ~~m~~ = **Example:** in a supercooled cloud with $S_e = 0.02$ ($s_e = 1.02$)

How long would it take for a drop to grow from 5×10^{-6} m to 1×10^{-3} m?

How long would it take an ice crystal to grow from 5×10^{-6} m to 1×10^{-3} m?

take : $T = 258\,K$, $P = 900\,hPa$

take $\rho_l^{-1}\alpha(T,P)^{-1} = 2.6\times10^{-11}\ m^2 s^{-1}$

$\rho_i^{-1}\beta(T,P)^{-1} = 2.4\times10^{-11}\ m^2 s^{-1}$

ratio of vp : $\frac{e_e(T)}{e_i(T)} = 1.15$. $\therefore\ s_i = s_e \times 1.15 = 1.02\times1.15 = 1.1730 \therefore S_i = 0.173$

Drop growth eq^n : $\boxed{a(t) = \sqrt{a_0^2 + 2(\rho_l\alpha)^{-1} S_e t}}$

Ice growth eq^n : $\boxed{a(t) = a_0 + \frac{8}{3\sqrt{3}}(\rho_i\beta)^{-1}\frac{m S_i t}{h}}$

$$\Rightarrow \boxed{\begin{array}{llll} t_{drop} = & 9.6\times10^5\,s & \text{or} & 11 \text{ days!} \\ t_{ice} = & \cancel{5.8\times10^3}\ 778\,s & \text{or} & \sim 10 \text{ mins!}\ \cancel{1.6\text{ hrs}} \end{array}}$$