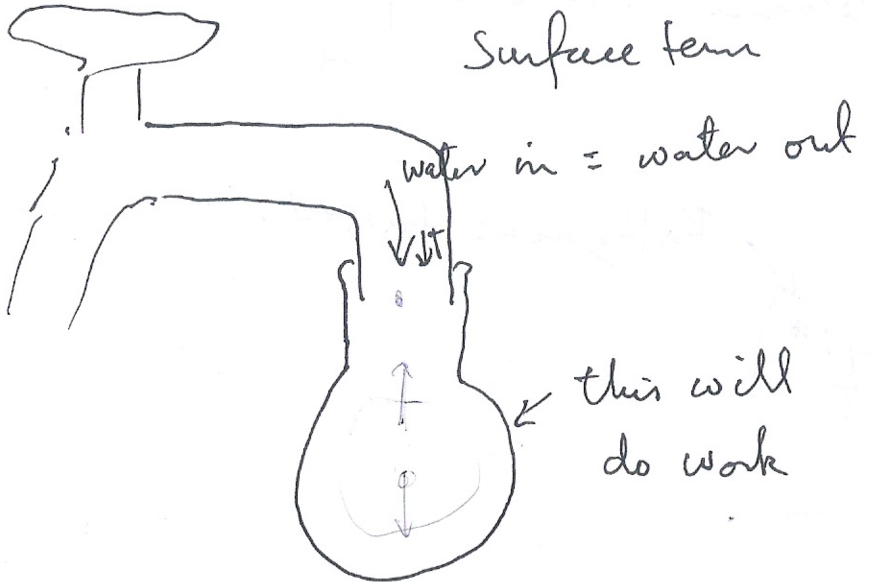
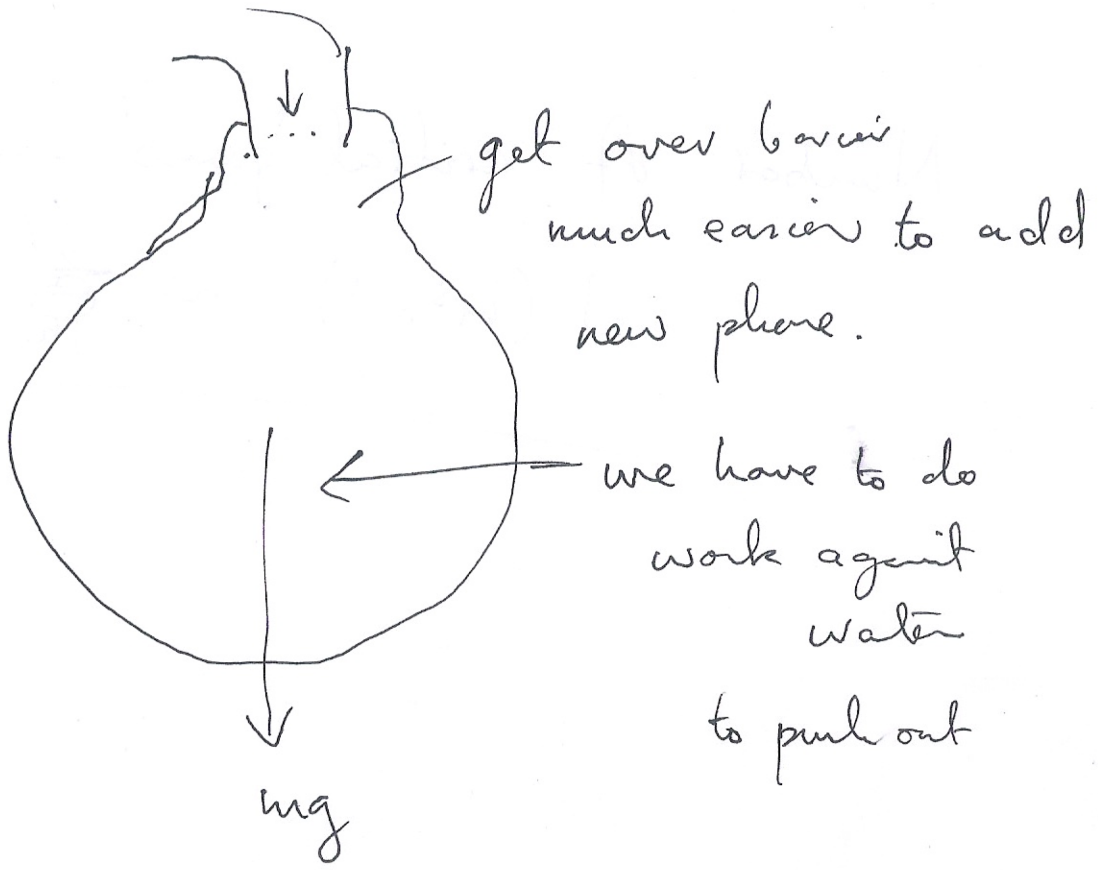
Nucleation (derivations not need)

Change in Gibbs free energy for ice particle growing in liquid

~~Chemical potential of ga~~

$$\Delta G_i = \underbrace{4\pi r_i^2 \sigma_{i/w}}_{\text{Surface term}} - \underbrace{\frac{4}{3}\pi r_i^3 \times \left(\frac{kT \ln S_i}{v_{ice}}\right)}_{\text{volume term}}$$





Critical radius, r^*, occurs at $\dfrac{\partial \Delta G_i}{\partial r_i} = 0$

$$r^* = \frac{2 v_{ice}\, \sigma_{i/w}}{kT \ln S_i}$$ at this point we have nucleation

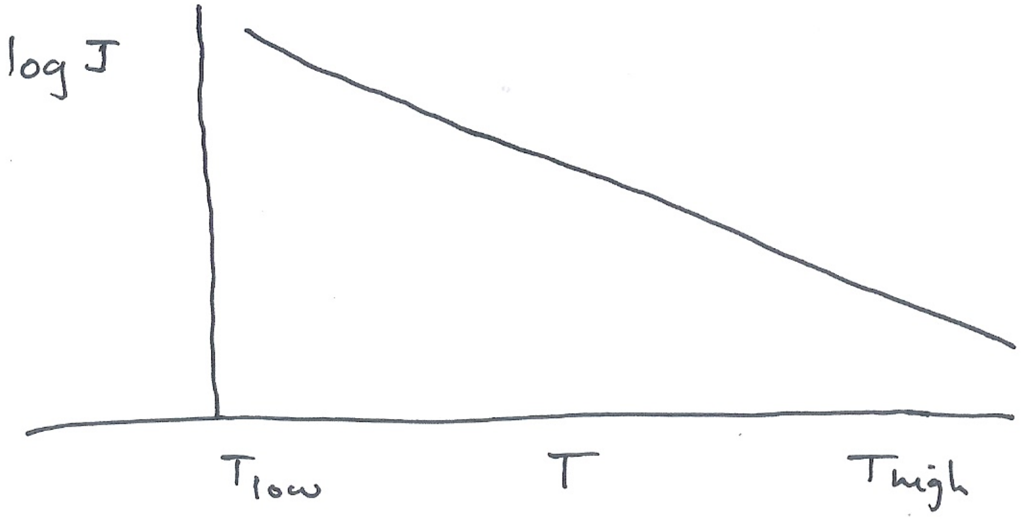
$$\Delta G^* = \frac{16\pi}{3} \frac{v_{ice}^2\, \sigma_{i/w}^3}{(kT \ln S_i)^2}$$ [this decrease with temp]

Number of critical 'germs' ... Boltzman factor.

$$J\ (m^3 s^{-1}) \sim \cancel{e}\, f_n\left(-\frac{\Delta G_i^*}{RT}\right)$$

Lecture 5.

Homogeneus nucleation of ice in supercooled water is a stochastic process that depends on volume of water present.



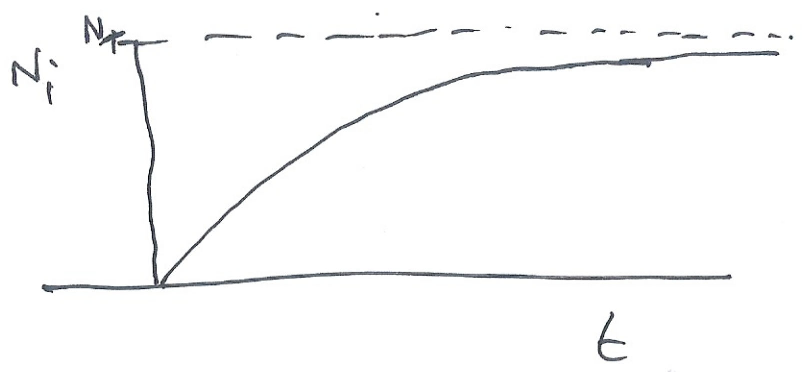
Define. nucleation rate, J [# $m^{-3} s^{-1}$]

$$\frac{dN_i}{dt} = J(T) \times V_d N_d$$

[N_i = number of ice crystals
N_d = number of drops
V_d = Volume of drop]

total particles = $N_T = N_I + N_d$

$$\underline{N_i(t) = N_T \left(1 - e^{-J(T) V_d t}\right)}$$



Heterogeneous ice nuclei (IN)

Stochastic

$$\frac{dN_i}{dt} = \omega(T)\, A\, (N_T - N_i)$$

$\omega(T)$: nucleation rate of ice on substrate $[\# m^{-2} s^{-1}]$

A = surface area of substrate

$\omega(T)$ is dependent on material

$$\Rightarrow \quad N_i(t) = N_T \left(1 - e^{\omega(T) A t}\right)$$

Singular

$$\frac{dN_i}{dT} = k(T)\, A\, (N_T - N_I)$$

$k(T)$ differential nucleus spectrum $[\# m^{-2} K^{-1}]$

$n_s(T) = \int_{T_0}^{T_c} k(T)\, dT \quad [\# m^{-2}]$

$$\Rightarrow \quad N_i(T) = N_T \left(1 - e^{n_s(T) A}\right)$$

Example :- a) how many pure drops (of 100) freezes in 10 seconds @ $-33°C$

drop size $= 30 \mu m$, $J(-33) = 10^{6} \# m^{-3} s^{-1}$

$V_d = 1.4 \times 10^{-14} \; m^3$

$\therefore N(10) = 100\left(1 - e^{10^{6} \times 1.4 \times 10^{-14} \times 10}\right)$

$= \underline{1 \times 10^{-5}}$!

b) $-38°C$? $J(-38) = 10^{16} \# m^{-3} s^{-1}$

$\therefore \underline{N(10) = 100}$