Lecture 2

Clausius - Clapeyron equation (all know).

$$\frac{de}{dT} = \frac{L_v}{T\Delta V}, \quad \Delta V = \frac{R_v T}{e} \quad [\text{ideal gas law}]$$

L_v = latent heat vaporisation

$$e_{sw} = 610.7\, e^{\left[\frac{L_v}{R_v}\left(\frac{1}{273.15} - \frac{1}{T}\right)\right]}$$

→ Saturation vapour pressure over liquid
why putting liquid in cloud chamber causes air to saturate.

$$e_{si} = 610.7\, e^{\left[\frac{L_s}{R_v}\left(\frac{1}{273.15} - \frac{1}{T}\right)\right]}$$

→ saturation vapour pressure over ice.
L_s = latent heat sublimation $= 2.8 \times 10^6\ J\,kg^{-1}$

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saturation ratio :-

$s_e = \frac{e}{e_{sw}}$ → saturation ratio ~~over~~ wrt liquid water

$s_i = \frac{e}{e_{si}}$ → sat. rat. wrt ice.

Supersaturation :-

$S_e = s_e - 1$
$S_i = s_i - 1$

} $S > 0$ slight required for cloud formation

Example :- you try?

$$W_s = \frac{\varepsilon e_s(T)}{P}$$

If I calculate the difference W_s at two levels that will give adiabatic LWMR — which do you take away from which?

$$W_{s1} = \frac{\varepsilon \times 610.7\, e^{\left[\frac{2.5\times10^6}{461}\left(\frac{1}{273.15} - \frac{1}{280.15}\right)\right]}}{900\times10^2}$$

$$\left[\ \varepsilon = \frac{R_a}{R_v} = 0.622\ \right]$$

$= ~~0.0074~~\ 6.93\times10^{-3}\ ~~7.36\times10^{-3}~~\ \mathrm{kg\,kg^{-1}}$

$$W_{s2} = \frac{\varepsilon \times 610.7\, e^{\left[\frac{2.5\times10^6}{461}\left(\frac{1}{273.15} - \frac{1}{279.65}\right)\right]}}{850\times10^2}$$

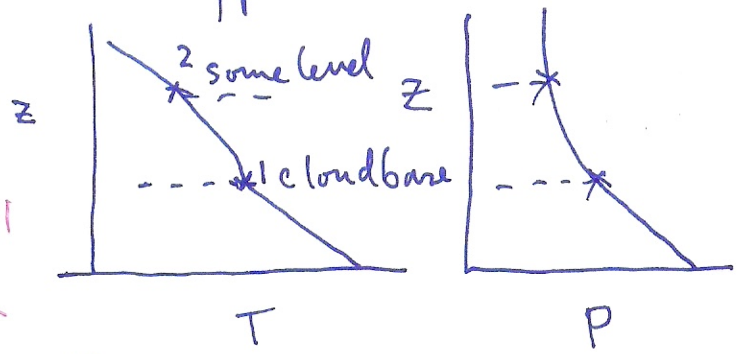
$= ~~6.17\times10^{-3}~~\ ~~6.41\times10^{-3}~~\ \mathrm{kg\,kg^{-1}}$ $\quad 6.7\times10^{-3}\ \mathrm{kg\,kg^{-1}}$

~~0.27×10⁻³ kg/kg~~

$\therefore\ W_{s1} - W_{s2} \approx ~~0.8\times10^{-3}~~$ $\quad 0.23\ \mathrm{g\,kg^{-1}}$

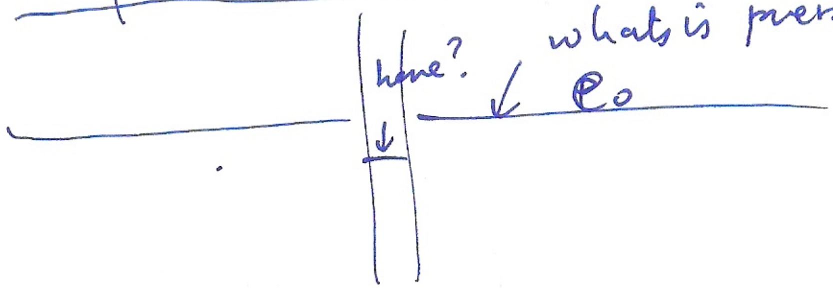
Also web page

Approx.



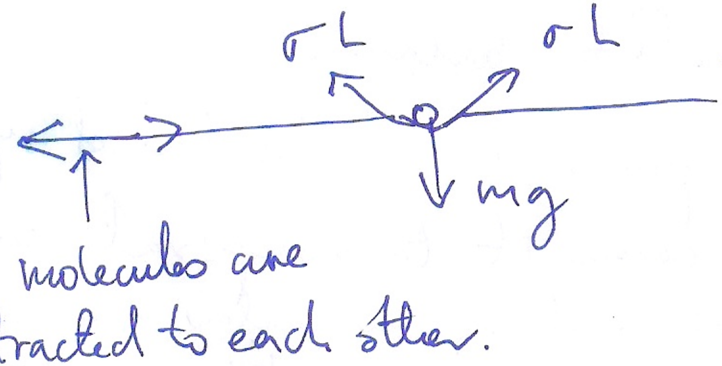
$$\Delta LWMR = W_{s,1} - W_{s,2}$$

Recap on flat surfaces Q 1. Why is water in tube below water line?



– take answers

Q 2.



Why does the pin stay afloat?
→ displaces water that is heavier than it?
Archimedes – No!
Steel is denser than water and the pin in on top of water.

Surface tension

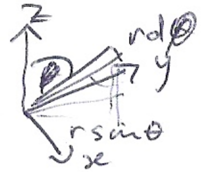
Q3

– Balloon

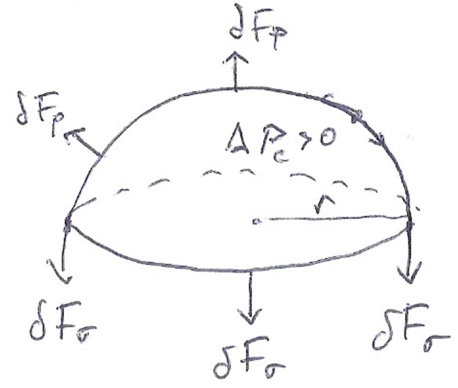
Any ideas on how I could calculate ΔP from shape? What would I need?

Derive it.

Consider a capillary tube - why? - turns out to be relevant.



$dA = r^2 \sin\theta \, d\theta \, d\varphi$



$\delta F_p = \Delta P_c \, r^2 \sin\theta \, d\theta \, d\varphi$

but we want component in z dir^n

$\delta F_{pz} = \Delta P_c \, r^2 \sin\theta \, d\theta \, d\varphi \times \cos\theta$

$$\therefore \; F_p = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \Delta P_c \, r^2 \sin\theta \cos\theta \, d\theta \, d\varphi$$

$$= \underline{\Delta P_c \pi r^2}$$

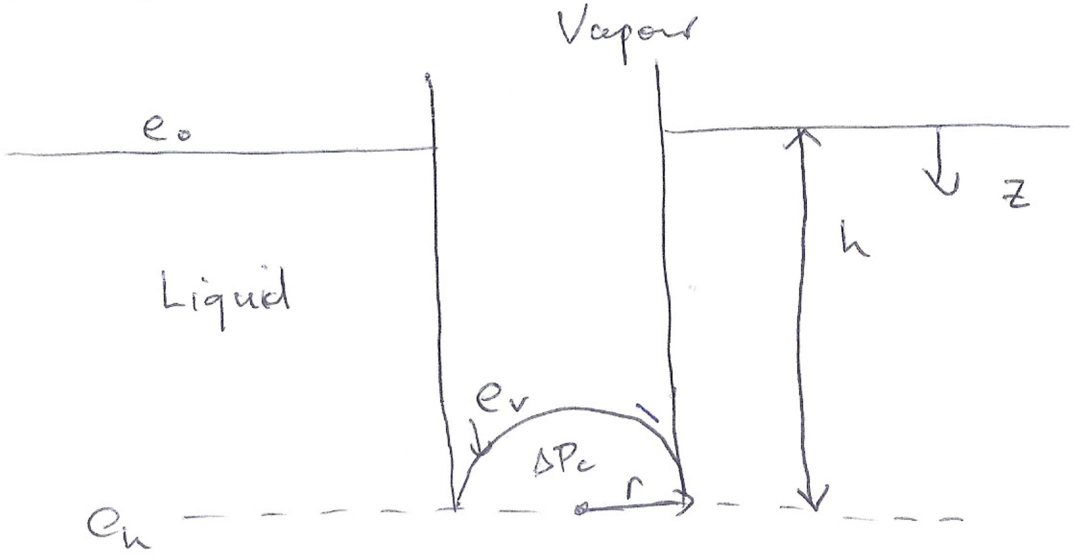
$$\underline{F_\sigma = 2\pi r \sigma}$$

$$\Delta P_c \not{\pi} r^{\not{2}} = 2 \not{\pi} \not{r} \sigma$$

$$\Delta P_c = \frac{2\sigma}{r}$$

[Pressure inside drop
Laplace pressure.]

Capillary tubes - why? it turns out to be relevant.



total pressure outside tube at h:

$$e_h = e_o + \rho_e g h \quad \text{①} \quad \left[\rho_e g h \text{ weight of liquid}\right]$$

total pressure at h inside tube :-

$$e_h = \cancel{e_o} + (e_v - \cancel{e_o}) + \Delta P_c \quad \text{—②}$$

hydrostatic relation for vapor :- $\left[\frac{de}{dz} = \frac{e}{R_v T} g\right]$

$$e_v = e_o \exp\left(\frac{gh}{R_v T}\right) \quad \text{—③}$$

~~$e_o + \rho_e g h = e_o \exp\left(\frac{gh}{R_v T}\right) + \Delta P_c$~~

Combine ① + ② & $e_o \approx e_v$

$$\cancel{e_o} + \rho_e g h = \cancel{e_v} + \Delta P_c$$

$$\Rightarrow \quad h \approx \frac{\Delta P_c}{\rho_e g} \quad \text{—④}$$

④ in ③ :

$$e_v = e_o \exp\left(\frac{\Delta P_c}{\rho_e R_v T}\right)$$

Sub ΔP_c :

$$e_v = e_o \exp\left(\frac{2\sigma}{\rho_e R_v T r}\right)$$

Kelvin's (aka William Thompson.) equation - weird?

Vapor pressure required to support small drops depends on size?

- Balloon analogy

Key points.

- Saturation vapour pressure over liquid (water)
- Saturation vapour pressure over ice
- Saturation ratio & super-saturation of water vapour
- Why size is important for the growth of aerosol particles into cloud drops – Kelvin's equation.