lecture 1

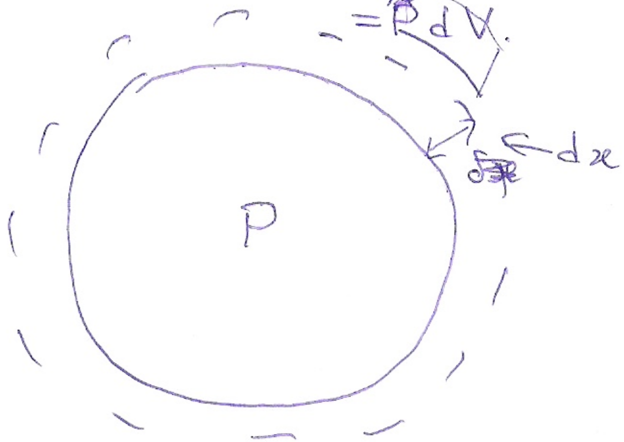
1st law TD:-

[c_v = specific heat constant vol]

$$dU \quad = \quad Q \quad - \quad W$$

[change in internal energy] [Heat added to system] [Work done by system]

$W = F \times dx$
$= \frac{P}{\cancel{A}} A dx$
$= P dV$



Consider 1 ~~mole~~ kg of air (parcel)

why? $\cancel{\frac{5}{2}} R_a dT = \cancel{\frac{5}{2}}(c_p - c_v) dT$ and $c_p = \frac{7}{2} R_a = \frac{7}{5} c_v$

$$c_v dT \quad = \quad \delta q \quad - \quad p\, dV \quad \text{——— (1)}$$

[heat capacity] [heat added] [Work done by system]

Ideal gas law: for 1 ~~mole~~ kg of air:

$P\cancel{V} = R_a T$ [$R_a = \cancel{8.314}$ 287 $J K^{-1}$ ~~mol~~ kg^{-1}]

$\therefore \cancel{V} = \frac{R_a T}{P}$ [v = volume per kg]

differentiate (quotient rule):

$$dV = \frac{P R_a dT - R_a T dP}{P^2} \quad \text{——— (2)}$$

Sub (2) in (1):

$$c_v dT = \delta q - R dT + \frac{R_a T dP}{} \quad \text{——— (3)}$$

Rearrange to find:

$$(R_a + c_v)\, dT = \delta q + \frac{R_a T}{P}\, dP$$

[Mayer's relation
$c_p - c_v = R_a$]

$c_p = 1005 \text{ J kg}^{-1}\text{K}^{-1}$

$$\therefore \quad c_p\, dT = \delta q + \frac{R_a T}{P}\, dP \quad \text{— (4)}$$

Adiabatic $\Rightarrow$ $dq = 0$

$$\frac{dT}{dP} = \frac{R_a T}{c_p P}$$

[Consider parcel rising from T_0, P_0 to T, P]

$$\Rightarrow \ln \frac{T_0}{T} = \frac{R_a}{c_p} \ln \frac{P_0}{P}$$

OR dry potential temperature [set $T_0 = \theta$, $P_0 = 1000\text{ hPa}$]

$$\theta = T\left(\frac{1000\text{ hPa}}{P}\right)^{R_a/c_p}$$

Conserved during dry adiabatic process

271·8K

Example ~~dust~~ chamber $T = 293\text{K}$, $P = 1300\text{ hPa}$ $\rightarrow$ $\theta = ?$ θ_1
$T = ?$ 271·8K $P = 1000\text{ hPa}$ $\rightarrow$ $\theta = ?$ θ_1

Moist air:-

Eqⁿ (4) with $dq = L_v dw_\ell$

$$c_p\, dT = \cancel{dq} + \frac{R_a T}{P} dP + L_v dw_\ell \quad [-L_v dw_s]$$

[latent heat vapourisation]

L_v = latent heat vap $2.5 \times 10^6\ \mathrm{J\,kg^{-1}}$

w_ℓ = liquid water mixing ratio $\mathrm{kg\,kg^{-1}}$

w_s = saturation vap. mixing ratio $\mathrm{kg\,kg^{-1}}$

$$\Rightarrow \ln\frac{T_0}{T} = \frac{R}{c_p}\ln\frac{P_0}{P} - \frac{L_v w_{s0}}{Tc_p} + \frac{L_v w_s}{Tc_p}$$

$\int \frac{L_v}{Tc_p} dw_s$ $\quad \int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

$T^{-1} \Rightarrow \ln T$

$\Rightarrow$ Choose $T_0 = \theta$, $P_0 = 1000\,\mathrm{hPa}$,

$$\Rightarrow \boxed{\theta_{q,sat} = T\left(\frac{1000\,\mathrm{hPa}}{P}\right)^{R_a/c_p} e^{\frac{w_s L_v}{Tc_p}}}$$

where $\theta_{q,sat} = T_0\, e^{\frac{L_v}{Tc_p} w_{s0}}$ @ $\cancel{P = 1000\,\mathrm{hPa}}$

[just a convention]

$\theta_{q,sat}$ is conserved during moist adiabatic process

Example cloud chamber $\quad T = 293\,\mathrm{K}$, $P = 1300\,\mathrm{hPa}$ $\Rightarrow \theta_{q,sat} = ?$ ← same

$w_s = ?$

$T = 293\,\mathrm{K}$ $\quad P = 1000\,\mathrm{hPa}$ $\Rightarrow \theta_{q,sat} = ?$ $\quad w_s$

What is w_s?

Saturation vapour mixing ratio $[\mathrm{kg\ kg^{-1}}]$

Ideal gas law for vapour.

$$e_s = \rho_{v,s} R_v T$$

[e_s = saturation vapour pressure Pa
$\rho_{v,s}$ = saturation vapour density $\mathrm{kg\,m^{-3}}$
R_v = specific gas constant water vap $461\ \mathrm{J\,kg^{-1}\,K^{-1}}$]

Ideal gas law for air:-

$$P = \rho_a R_a T$$

[P = total pressure
ρ_a = density of air $\mathrm{kg\,m^{-3}}$
R_a = specific gas constant air $287\ \mathrm{J\,kg^{-1}\,K^{-1}}$]

mixing ration

$$\therefore \frac{\rho_v}{\rho_a} = \frac{e_s}{\cancel{T} R_v} \times \frac{\cancel{T} R_a}{P}$$

$$= 0.622 \frac{e_s(T)}{P}$$

How calculate LWCR in cloud?

w_s depends on P & T.

$$\Delta LMR = w_s(T_1, P_1) - w_s(T_2, P_2)$$

Key points:-

θ conserved during dry adiabatic process

w_s depends on $\underline{\underline{T}}$ & P. (how ~~p~~ much water vapour air can hold)

$\theta_{q,sat}$ conserved during moist adiabatic process.

Need <u>particles</u> to form a cloud! Aerosol particles.