Viscous dissipation and apparent wall slip in capillary rheometry of ice cream

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1. Ice cream: industrial process rheology

The appeal of ice cream is closely related to its rheology and microstructure; both are carefully developed during the production process. For example, the surface lustre and sharkskin fracture apparent in Figure 1 are indicative of wall slip and yield stress effects during serving.

However, the rheology of ice cream during processing is very different from that of the final product after it has been hardened. The system is complex since ice cream is a multi-phase, viscous and temperature sensitive material: viscous dissipation at the wall could cause sufficient heating to reduce the viscosity of the fluid near the wall and therefore generate apparent slip. Furthermore, ice cream cannot be readily simulated in the laboratory and tests must be carried out on pilot scale production equipment in order to be realistic. This study has attempted a rigorous separation of the effects of wall slip and viscous dissipation.

Figure 1. Ice cream (source: iStockphoto).

2. Viscous dissipation and apparent wall slip

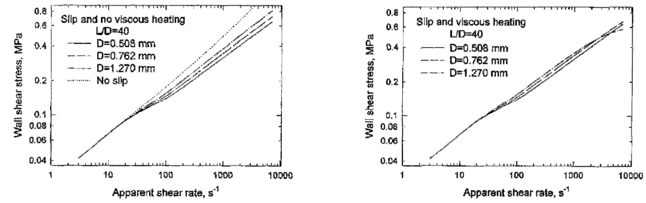
Many foods suspensions display visco-plastic behaviour and exhibit apparent wall slip due to a depletion of solids near the wall, resulting in a highly sheared low viscosity layer. Assuming wall slip is often useful, but the limits to its application are not known and can cause serious errors when interpreting data from rheological measurements. Mooney's classical technique of accounting for apparent wall slip often fails, and untested variants to account for non-standard cases (e.g. that by Jastrzebski) have prospered[1].

Mooney's equation for slip:
(Q – flow rate, R – radius, V_s – slip velocity, τ_w – wall shear stress)

$$\frac{4Q}{\pi R^3} = \frac{4}{R}V_s + \frac{4}{\tau_w^{3}}\int_{\tau_y}^{\tau_w} f^{-1}(\tau)\,\tau^2\,\mathrm{d}\tau$$

Viscous dissipation can also cause phenomena which are analogous to the above 'wall depletion effects in flow'. This mechanism has attracted significant attention in the rheology of highly viscous synthetic polymer melts[2]. In this instance, the energy dissipated against the fluid while pumping it increases the temperature; the highest shear rates are obtained near the wall, resulting in a higher wall temperature, lower viscosity, and therefore very high shear rates in this region.

Whilst both effects result in high shear near the wall, slip is more significant at smaller pipe diameters and viscous dissipation is more significant at larger pipe diameters. This is elegantly illustrated for polymer melts in Figure 2, where the effect of viscous dissipation becomes significant for the larger pipe at shear rates approaching 10^5 s^{-1}.



(a) Without viscous dissipation **(b) With viscous dissipation**

Figure 2. Calculated flow curves for polymer melts with wall slip[2].

3. Pilot plant pipe rheometry

A total of 224 different experimental flows were studied. The apparent viscosities for all of these flows are shown against average core temperature in Figure 5. Order of magnitude variations in apparent viscosity were found, with scatter being greater at lower temperatures. Initial analysis of the ice cream data sets using classical Mooney analysis indicated that significant slip effects were present[3]. The bulk constitutive behaviour could be modelled as a power law fluid, viz. $\tau = K\dot{\gamma}^n$ with n ~ 0.5, as shown in Figure 3.

This fortuitous result allowed us to evaluate the likely effect of viscous heating on the industrial test system using the semi-analytical model of Bird[4] for adiabatic flow of a power law fluid along a tube. The impact of viscous dissipation on the pilot plant trials was estimated by using the analytical model to evaluate the temperature rise at the wall calculated at the end of the test section for the geometries and conditions employed in the tests. The results for a representative test are presented as a parameter space plot in Figure 4.

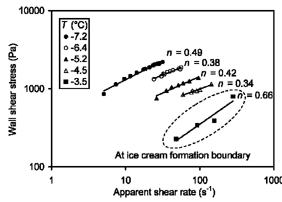


Figure 3. Flow curves at various temperatures.

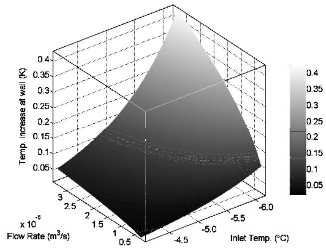


Figure 4. Extent of viscous heating at the wall predicted by the analytical model.

A wall temperature rise of 0.1 K was used as a threshold for filtering out data points where viscous dissipation could be occurring - Figure 5 shows that this filtering removes 79 points, primarily in the region of lower temperature, where the associated high viscosity is likely to cause viscous dissipation. The remaining 145 data points show a more consistent trend, through which the fit for an exponential trend line is illustrated. Following this selection, the remaining data did not show consistent signs of wall slip and could be rheologically characterized as shown in Figure 6.

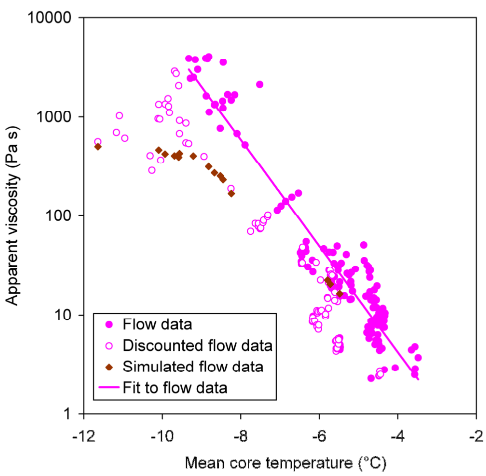
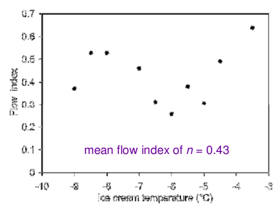
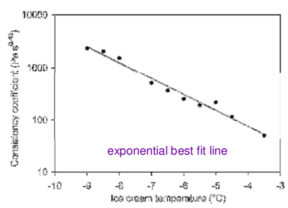


Figure 5. Apparent viscosity against mean core temperature.



(a) Fitted flow indices



(b) Fitted consistency coefficients

Figure 6. Ice cream flow behaviour for filtered data sets.

4. CFD simulation

A finite element based method CFD package was used for the prediction of the flow and heat transfer behaviour of the ice cream flowing in the pipes.

A set of experimental cases were simulated where significant viscous dissipation was expected on the basis of the analytical solution. The results plotted on Figure 5 exhibit a distinct levelling off in apparent viscosity and deviation from the simple exponential dependency on temperature evident in the filtered experimental data. The agreement between the experimental and simulation results for this case of strong viscous dissipation was considered reasonable, particularly given that a power law index of 0.5 was used in the simulations whereas the observed values varied around this.

Figures 7 and 8 show the simulated temperature and velocity development along the pipe when viscous dissipations is significant using temperature dependent ice cream properties.

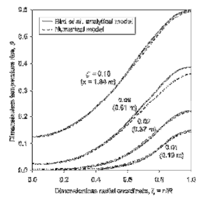


Figure 7. Comparison of predicted temperature profiles across the pipe for the adiabatic wall case.

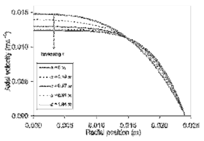


Figure 8. Numerical prediction of velocity profiles along the pipe.

5. Conclusions

Application of theoretical models of viscous dissipation reveal that this was significant in over one third of the flows, despite low shear rates of 10-100 s^{-1}. A new numerical implementation of these models involving temperature dependent enthalpy, thermal conductivity and apparent viscosity showed how such flows tend towards the characteristics of apparent wall slip in cases where wall slip might not be expected. Analysis of the data set filtered to remove significant viscous dissipation showed that a power law model with a consistency coefficient varying exponentially with temperature represented the ice cream reasonably well[5].

References

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