Applications of the model theory of fields with operators

Abstracts of Monday 17th June

Jonathan Pila. *"Independence of CM points in elliptic curves"*

I will speak about joint work with Jacob Tsimerman. Let E be an elliptic curve parameterized by a modular (or Shimura) curve. There are a number of results (..., Buium-Poonen, Kuhne) to the effect that the images of CM points are (under suitable hypotheses) linearly independent in E. We consider this issue in the setting of the Zilber-Pink conjecture and prove a result which improves previous results in some aspects.

Michael Wibmer. *"Model theory of proalgebraic groups"*

The model theory of profinite groups has been studied since the eighties. Key facts are that the theory of the absolute Galois group is interpretable in the field and that certain saturated models correspond to free profinite groups. In this talk we will report on recent progress in adapting these ideas to differential fields. In particular, we will introduce free and saturated proalgebraic groups and discuss an axiomatization of proalgebaic groups. Joint work with Anand Pillay.

Tamara Servi. *"Transserial asymptotic expansions, oscillatory integrals and o-minimality"*

Logarithmic-exponential series (or transseries) appear naturally in the study of the asymptotic behaviour of certain real functions (as it will be illustrated in J.-P. Rolin's and P. Speissegger's talks). I will discuss several notions of asymptotic expansion in the logarithmic-exponential scale, and one which is particularly suitable for the asymptotic analysis of parametric integrals such as exponential periods and Fourier transforms of subanalytic functions (joint work with R. Cluckers, G. Comte, D. Miller and J.-P. Rolin). Motivation and methods come from o-minimality, classical asymptotic analysis and number theory.

Jean-Philippe Rolin. *"Transserial solutions of Abel's equation"*

A transseries is a formal series whose monomials involve exponentials and logarithms. Some aspects of the book of M. Aschenbrenner, L. van den Dries and J. van der Hoeven, in which they study the algebra and model theory of the differential field of transseries, are directly related to well-known results in dynamical systems. We show how to extend these results in the setting of Dulac's problem, in which transseries appear as asymptotic expansions of real functions.

Abstracts of Tuesday 18th June

Ivan Tomasic . *"A topos-theoretic view of difference algebra"*

Abstract difference and differential algebra were founded by Ritt in the 1930s as the study of algebraic structures endowed with difference/differential operators. This approach has a long and productive history, but attempts to develop methods of homological algebra within this context quickly reach insurmountable obstacles.

We will show that difference algebra is best viewed as the algebra of objects internal to the topos of difference sets. Indeed, using the methods of topos theory and categorical logic, we can elevate the study of difference algebraic geometry to the level of classical algebraic geometry. Time permitting, we will explain how to treat the difference-differential case in this context.

Paola D'Aquino. *"Zero sets of exponential polynomials"*

I will review the algebra of exponential polynomials and properties of the set of roots.

Rachael King. *"Galois covers of difference schemes'*

We will discuss actions of difference groups on difference schemes and define Galois covers of difference schemes. After comparing these to notions appearing in papers on difference Galois stratification by I. Tomasic, we will develop the corresponding Galois theory.

Jonathan Kirby. *"Local interdefinability of elliptic functions'*

Considering o-minimal expansions of the real field by certain restricted analytic functions, one can ask which other analytic functions are (locally) definable. Jones, Servi and I answered the question for exponentiation and Weierstrass $\wp$-functions, showing that local definability corresponds to isogeny of the relevant elliptic curves. I will discuss this and recent work with Jones and Schmidt, extending the results to other elliptic functions such as the Weierstrass ζ-functions.

Abstracts of Wednesday 19th June

Angus Macintyre. *"Elementary Equivalence and Isomorphism for rings A_K, where K is a number field. (Joint with J.Derakhshan)."*

Though all number fields K are undecidable,all adele rings A_K are decidable,and definability is well-understood for the individual A_K. Thus K is certainly not interpretable in A_K. But for nearly 90 years number theorists have studied the

question of whether the isomorphism type of A_K determines that of K, or at least some fundamental invariants of K, such as its zeta function. Negative answers are known, involving serious group theory and algebraic number theory.However, strong positive results are known, too, for example that the isomorphism type of A_K determines K up to finitely many possibilities. We give a systematic treatment in model-theoretic terms, and show that for adele rings elementary equivalence and isomorphism coincide. We discuss briefly axioms for individual adele rings, and discuss decidability results for various infinite classes of adele rings .

Gleb Pogudin. *"Primitive Element Theorem for fields with commuting derivations and automorphisms."*

Primitive Element Theorem is a classical tool in field theory and symbolic computation. In 1942, Kolchin proved an analogous theorem for partial differential fields. Similar generalization has been obtained by Cohn in 1965 for difference fields with one automorphism. These theorems guarantee that if an extension $F \subset E$ is finitely generated and algebraic in an appropriate sense and the ground field F is "nonconstant", then the extension can be generated by a single element. These generalizations played an important role in differential/difference algebra and its applications.

Both theorems by Kolchin and Cohn imposed an extra condition on the ground field F of being "nonconstant" that makes them not applicable to many interesting extensions coming from autonomous differential/difference equations or algebraic varieties equipped with a vector field or an automorphism. In 2015, I have partially resolved this issue by strengthening Kolchin's theorem in the case of one derivation so that the condition that F contains a nonconstant was replaced by a natural condition that E contains a nonconstant (otherwise, the derivation would be zero).

In this talk, I will describe my recent result that generalizes the primitive element theorems by Kolchin, Cohn, and myself in two directions

- the existence of a primitive element is established for fields with any number of derivations and automorphism commuting with each other (this includes, for example, partial difference and differential-difference fields);
- no extra condition on the ground field is imposed.

Abstracts of Thursday 20th June

Patrick Speissegger. *"Quasianalytic Hardy fields"*

(Joint with Zeinab Galal and Tobias Kaiser)

As another example of working with transserial asymptotic expansions–similar to Tamara's and Jean-Philippe's talks–I explain how to construct a quasianalytic Hardy field extending both the Hardy field of $(\mathbb{R}_{an}, \exp)$ and Ilyashenko's class of almost regular germs.

Sebastian Eterovic. *"Systems of Polynomial Equations Involving the j Function (joint work with Sebastin Herrero)"*

Inspired by the work done for systems of exponential polynomial equations, we study systems of polynomial equations involving the j function. We show general cases in which these systems can be solved, and then we show certain situations in which the modular Schanuel conjecture implies that these systems have generic solutions.

Alex Wilkie. *"PARAMETERIZATION"*

I will give an introduction to the various o-minimal parameterization theorems that have been established over the last ten years or so with particular focus on the issue of uniformity within definable families.