Model-theoretic methods in number theory and algebraic differential equations

Abstracts of Friday Aug 3rd

Dugald Macpherson. *"Measurable structures and generic automorphisms"*

I will discuss notions of measurable structure (work with Elwes and Steinhorn, more recent variants with Anscombe, Steinhorn and Wolf). I will also discuss the Chatzidakis-Pillay generic automorphism construction generalising ACFA, and the phenomenon of when the fixed point set satisfies notions of measurability.

Giuseppina Terzo. *"Complex exponentation and Zilber fields"*

Assuming Schanuel?s Conjecture, we prove that for any variety V of dimension n contained in $\mathbb{C}^n \times (\mathbb{C}*)^n$ over the algebraic closure of the rational numbers, and under some natural hypothesis, there exists a generic point in V of the form $(a, \exp(a))$. This result implies many cases of Zilber?s Conjecture.

Harry Schmidt. *"Iterated polynomial equations "*

In joint work with Gareths Boxall and Jones we study polynomials of the form $P^n(x) - P^n(x_0)$ where P^n is a polynomial defined over a number field iterated n times and x_0 is a fixed algebraic number. We show that this polynomial has many large irreducible factors. This result may be seen as a step towards a conjecture of Jones and Levy on equations of the form $P^n(x) - x_0$. Our approach uses an "improved Pila-Wilkie" counting result for graphs of certain analytic functions.

Martin Bays. *"Pseudo-exponential maps"*

Zilber conjectured that the complex exponential field $(\mathbb{C}; +, *, \exp)$ is quasiminimal - any definable subset is countable or has countable complement. In part as an approach to proving this, Zilber showed how to construct a quasiminimal exponential field ("pseudo-exponentiation") which is isomorphic to the complex exponential field if Schanuel's conjecture and a certain "existential closedness" conjecture are true.

I will talk on recently published work with Jonathan Kirby exploring variants of this construction. In particular we show that performing a "generic" form of the construction allows one to prove quasi-minimality for (a wider class of) exponential maps assuming only a substantially weakened form of the existential closedness conjecture, eliminating Schanuel's conjecture entirely.

Patrick Speissegger. *"Limit cycles of planar vector fields, Hilbert's 16th problem and o-minimality"*

Recent work links certain aspects of the second part of Hilbert's 16th problem (H16) to the theory of o-minimality. One of these aspects is the generation and destruction of limit cycles in families of planar vector fields, commonly referred to as "bifurcations". I will outline the significance of bifurcations for H16 and explain how logic–in particular, o-minimality–can be used to understand them well enough to be able to count limit cycles.

Abstracts of Saturday Aug 4th

Anand Pillay . *"Logarithmic differential equations on nonconstant semiabelian varieties"*

In connection with Ax-Lindemann-type problems for families of semiabelian varieties, Bertrand and I made some differential Galois-theoretic conjectures. I will discuss these conjectures, progress on them, as well as a restatement in a joint paper with Leon-Sanchez.

Margaret Thomas . *"Definable topologies and definable compactness in o-minimal structures"*

We consider various properties of topological spaces definable in o-minimal structures, and related properties of definable 'directed sets'. From our main result we derive the equivalence of different notions of definable compactness for such spaces (at least if the underlying structure satisfies certain further conditions, such as expanding a group, or a field) and identify an appropriate notion of 'definable first countability'. This is closely related to an ongoing project to study the classification of definable topological spaces. Joint work with Pablo Andujar Guererro and Erik Walsberg.

Zoé Chatzidakis. *"Definable subgroups of the additive group in existentially closed difference fields"*

This talk will survey some results about definable subgroups of $\mathbb{G}_a$ in models of ACFA. In characteristic 0, one knows that all such subgroups are internal to the fixed field, and therefore have a rich structure. In positive characteristic, there are however modular subgroups, and we will discuss their properties. In contrast with definable subgroups of semi-abelian varieties, they are not stably-embedded nor stable, at least in order 1.

Joel C. Ronnie Nagloo. *"The Ax-Lindemann-Weierstrass with derivative and the genus 0 Fuchsian Groups: Part I"*

The works of Pila and later Freitag and Scanlon, give the Ax-Lindemann-Weierstrass with derivatives for the Hauptmoduls of arithmetic subgroups of $PSL_2(\mathbb{Z})$. A challenge has been to prove similar transcendence results for the Hauptmoduls of all Fuchsian groups of genus zero. In this talk we will talk about recent progress towards the resolution of those problems. This is report of joint work between Guy Casale, James Freitag and Joel Nagloo.

Part I: I will introduce the main notions and also explain what is meant by the (differential) Ax-Lindemann-Weierstrass in this context. I will point out the relevance of the model theory of differentially closed fields of characteristic 0 and give a criterion for proving strong minimality/irreducibility of Schwarzian equations.

James Freitag. *"The Ax-Lindemann-Weierstrass with derivative and the genus* 0 *Fuchsian Groups: Part II"*

Part II: (see introductory text in the abstract of Nagloo's Part I) In recent years, there has been a surge of interest in ALW-type transcendence theorems, in part because of their application to so-called special points conjectures. Roughly speaking, in a special points conjecture, one attempts to characterize the algebraic varieties which contain a Zariski-dense set of some collection of arithmetically interesting special points (e.g. CM-points in moduli of abelian varieties, torsion points in groups, or Hecke orbits). Our ALW-theorem, together with some additional input from the model theory of differential fields, can be used to prove some cases of the André-Pink conjecture (and generalizations).

Abstracts of Sunday Aug 5th

Françoise Point. *"Topological large fields, their generic differential expansions and transfer results"*

We start with a theory T of topological fields admitting quantifier elimination (in a relational expansion $\mathcal{L}$ of the theory of fields). Under natural hypotheses and in particular that topological fields satisfy a property of largeness, and assuming they are endowed with a definable topology, it is known that the class of existentially closed expansions to differential fields are models of a theory T_D^* and that T_D^* admits quantifier elimination in $\mathcal{L}_\delta$ (the language $\mathcal{L}$ to which we add the derivation δ). Note that in any model of T_D^*, we get a dense pair of models of T.

For instance if one starts with the class of real-closed fields, one obtains the class of closed ordered differential fields; an axiomatization known as CODF was given by M. Singer.

We will first review a number of known transfer results between T and T_D^* and their consequences for the theory of dense pairs of models of T. Then we will concentrate on elimination of imaginaries. In the case of CODF, there are now several proofs, for instance one using the description of definable types. We will show transfer of elimination of imaginaries between T and T_D^*, using a topological argument due to M. Tressl in the case of CODF.

This is a joint work with Pablo Cubides Kovacsics (Caen).

Moshe Kamensky. *"Fields with free operators in positive characteristic"*

Moosa and Scanlon defined a general notion of "fields with operators", that generalises those of difference and differential fields. In the case of "free" operators in characteristic zero they also analysed the basic model-theoretic properties of the theory of such fields. In particular, they showed in this case the existence of the model companion. In positive characteristic, they provided an example showing that the model companion need not exist. I will discuss work, joint with Beyarslan, Hoffman and Kowalski, that completes the description of the free case, namely, it provides a full classification of those free operators for which the model companion exists. If time permits, I will discuss additional properties, such as quantifier elimination.

Gareth Boxall. *"Points on a curve with a power on a curve"*

Let C_1 and C_2 be two irreducible closed algebraic curves in $\mathbb{G}_m^N$, for $N \geq 3$. Levin asked about points on C_1 which have a non-trivial power on C_2. Bays and Habegger addressed this question in the case where $C_1 = C_2$ and the curve is geometrically irreducible and defined over a number field. They gave two approximations to a conjecture which predicts only finitely many such points provided C is not contained in a proper algebraic subgroup of $\mathbb{G}_m^N$. We consider the problem of extending some of their work to the setting where $C_1 \neq C_2$.

Angus Macintyre . *"TBC and TBA"*