Topics in Geometric Semigroup Theory

University of York

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Abstracts

Monday

Minicourse: Combinatorial inverse semigroup theory, an introduction

John Meakin
University of Nebraska-Lincoln

In the minicourse, we will introduce inverse semigroups, and review the some of the history in developing an analogue of combinatoiral/geometric group theory in the context of inverse semigroups. We will cover free inverse semigroups and inverse semigroup presentations, as well as Stephen's approach to solving the word problem using inverse automata and Schützenberger graps. We will in particular present some results about the word problem for one-relator inverse semigroups, special one-relator inverse semigroups and Adian inverse semigroups. Other topics we may cover (time allowing) are Schein's theory of actions of inverse monoids by partial functions, and how that relates to immersions of graphs and higher dimensional Δ-complexes. Lastly, we will talk about graph inverse monoids and their relevance in the theory of Leavitt path algebras.

Membership problems in one-relator groups and the word problem for one-relator inverse monoids

Robert Gray
University of East Anglia

It is a classical result of Magnus proved in the 1930s that the word problem is decidable for one-relator groups. It is natural to investigate other one-relator algebraic structures. One-relator monoids were studied in the 1960s and 70s by Adjan, Oganessian, and Lallement, with the word problem shown to be decidable in a number of important cases. It remains a longstanding open problem whether the word problem is decidable for all one-relator monoids. A natural class of algebraic structures lying between monoids and groups is that of inverse monoids. The word problem for one-relator inverse monoids was first investigated by Margolis, Meakin and Stephen in 1987. Later Ivanov, Margolis and Meakin (2001) proved several important results that relate the word problem in one-relator inverse monoids to the word problem for one-relator monoids, and also to a decision problem for one-relator groups called the prefix membership problem. In this talk I will speak about some recent results on membership problems in one-relator groups and the word problem for one-relator inverse monoids, and connections between them.

Subgroups of special inverse monoids

Mark Kambites
University of Manchester

I will report on ongoing joint work with Robert Gray, studying the structure of finitely presented special inverse monoids, with a particular focus on the group H-classes.

A Lyndon's Identity Theorem for one-relator monoids

Benjamin Steinberg
City College of New York

Magnus solved the word problem for one-relator groups in the early 1930s. This spurred quite a bit of work into the study of one-relator Lie Algebras, rings and monoids. Adian and his school made a number of breakthroughs on the word problem for one-relator monoids in the sixties and seventies, but the problem remains wide open. In 2000 Kobayashi asked whether the word problem for one-relator monoids can be solved using the theory of finite complete rewriting systems. A necessary condition for a monoid to have a finite complete rewriting system is the homological finiteness condition FP_∞. Kobayashi asked in 2000 whether every one-relator monoid is of type FP_∞. Note that one-relator groups are of type FP_∞ as a consequence of Lyndon's identity theorem.

In this talk we sketch some techniques in the the proof that all one-relator monoids are of type FP_∞. The main technique involves constructing actions of monoids on contractible CW complexes. Adian's machinery from his work on the word problem is reinterpreted from a more geometric viewpoint to build these complexes.

This is joint work with Robert Gray from the University of East Anglia.

Tuesday

Decision problems in word-hyperbolic monoids

Carl-Fredrik Nyberg-Brodda
University of Manchester

Hyperbolic groups, first introduced by Gromov in 1987, form one of the cornerstones of geometric group theory. The definition of hyperbolic groups exploits the fact that the coarse geometric properties of the Cayley graphs of groups are rigid enough to encode much algebraic information. However, this is generally not the case for semigroups — here, the geometric properties are far too weak, and something more rigid is necessary. Gilman in 2002 gave a language-theoretic definition of hyperbolicity — henceforth word-hyperbolicity — in groups, which can be shown equivalent to the usual geometric definition. This definition extends in a natural way to semigroups, and was explored by Duncan & Gilman in 2004. In my talk, I will explore some lines of inquiry into word-hyperbolicity in special classes of semigroups. I will present some of my recent results on the subject, including the fact that the Diophantine problem — "solving equations" — can be undecidable in word-hyperbolic monoids. This is in contrast to the case of hyperbolic groups, where the problem is decidable (Dahmani & Guirardel 2010).

Minicourse: C*-algebras, their diagonals and (inverse) semigroup actions

Diego Martínez
Mathematical Institute, WWU Münster

In this minicourse we will introduce C*-algebras, and how they relate to (inverse) semigroups. A C*-algebra is an complex Banach algebra equipped with an involution * such that $||a^*a|| = ||a||^2$. When the norm $||\cdot||$ satisfies this last property it is usually called a *C*-norm*. C*-algebras are a wide and active research area, and encompass topics ranging from functional analysis to group theory.

During the talk, there shall be a special emphasis on examples, and how to construct large classes of C*-algebras. Of particular interest will be those examples coming from inverse semigroup actions on compact Hausdorff spaces, and the C*-algebras these actions generate. As we will see, these C*-algebras usually have a *diagonal*, that is, some extra structure that allows us to relate the properties of the actions to properties of the C*-algebra.

Lastly, this minicourse is self-contained, and no special knowledge of functional analysis will be required.

Inverse semigroups of asymptotical dimension 0 and their uniform Roe algebras

Diego Martínez
Mathematical Institute, WWU Münster

It is well known, and goes back to 1940's, that a countable discrete group is canonically equipped with a proper and right invariant metric. In this talk, we will prove the analogous result for inverse semigroups. In particular, given S we will equip it with a uniquely defined extended metric that is both proper (in a suitable sense) and right sub-invariant, and prove that such a metric is unique. Given such a metric, we will characterize the semigroups that have asymptotical dimension 0 (in the sense of Gromov), that is, those semigroups that are the simplest from a coarse geometric point of view. As it turns out, these are precisely the locally finite inverse semigroups, and their uniform Roe algebras enjoy some particularly strong approximation properties. This is based on joint work with Yeong Chyuan Chung and Nóra Szakács.

Equilibrium states of C*-algebras associated to right LCM monoids

Nadia Larsen
University of Oslo

C*-algebras associated to monoids provide constructions where features of the monoid reflect into analytic properties of interest in operator algebras. This kind of interplay is prominent in the study of equilibrium, or KMS states, for C*-dynamical systems, which are pairs consisting of a C*-algebra and a one-parameter automorphism group. In this context, KMS states are positive linear functionals on the C*-algebra in question that obey a certain trace-like condition involving a twisting of elements by the one-parameter group. For suitable classes of C*-dynamical systems associated to right LCM monoids, a KMS state is characterized by a positivity condition encoded in a system of inequalities. If the monoid admits a generating set, it is of interest to reduce the characterisation of positivity to a smaller number of inequalities depending on subsets of generators. For Noetherian monoids, this reduction can be formulated by using a combinatorially finite tree. We illustrate the case of finite-type Artin monoids and right-angled Artin monoids. The talk is based on joint works with Afsar, Neshveyev, Gazdag and Laca.

Wednesday

Approaching coarse geometry through inverse semigroups

Martin Finn-Sell
University of Vienna

In this talk I will explain how coarse geometric methods can be rephrased using the language of partial bijections, try to explain the way this developed through the literature without semigroups, and how making the connections explicit can yield some very cool results elsewhere. I'll do this mostly on a low level by working through some examples, and talking about these ideas fit into the bigger picture of two different (related) fields - index theory and classification of C*-algebras.

A near glimpse of near group actions

Yves de Cornulier
CNRS, University Lyon 1

A near action of a group on a set is defined in the same way as an action, except that one allows indeterminacies on finite subsets. I will discuss this notion and various examples. Notably, I will address the issue of realizability: what are obstructions for a near action to be induced by an action? For instance, when the acting group is $\mathbb{Z}^2$, this essentially amounts to the following: given two permutations f, g of a set such that the commutator $[f, g]$ has a finite support, can we modify f and g on finite subsets to get commuting permutations f', g'?

Minicourse: Amenability of semigroups

Mark Kambites
University of Manchester

I will give a basic introduction to amenable groups and semigroups, starting at an elementary level, and with a focus on geometric aspects and the finitely generated case.

Minicourse: Lectures on near group actions

Yves de Cornulier
CNRS, University Lyon 1

I will introduce in more detail near actions, after the separate talk, and various notions of realizability. Notably, I will introduce the notion of completability: a near action is completable if it arises as a near subaction of an action. In particular, I will explain why a near action is completable if and only if it is realiable as a partial action. I will also describe further examples of near actions, notably in the context of near automorphisms of trees. For instance, some groups such as Thompson's group V or Neretin's group can be defined as centralizers of suitable elements in a monoid of 'near maps'.

Thursday

Topological inverse monoids and full groups

Alejandra Garrido
Universidad Autónoma de Madrid

It has been known for some time that full groups of inverse monoids acting on Cantor sets (or ample groupoids) are a source of finitely generated infinite simple groups with interesting properties. They are then reasonably expected to yield examples of simple locally compact totally disconnected groups that are compactly generated. The first question is how to topologise these full groups to make them locally compact. It so happens that one should induce the topology from one on the inverse monoid whence the full group is obtained, which must satisfy certain conditions. If the inverse monoid is a group, the condition on its topology is a consequence of an elementary result on topological groups. Despite there seemingly being no analogue of this general result for topological inverse monoids, there are sufficient conditions that can be imposed to obtain the desired topology. This is a key step to obtaining locally compact totally disconnected groups that are simple and compactly generated.

No knowledge of topological groups or topological inverse monoids will be assumed. Examples will be given.

(An invitation to) reflection monoids

Brent Everitt
University of York

The theory of reflection groups is a venerable subject, with tentacles reaching into Lie theory, algebraic geometry, geometric topology,... The theory of hyperplane arrangements, which partially overlaps that of reflection groups, has emerged as an important field in algebraic combinatorics. A reflection group and a hyperplane arrangement can be bought together into a single algebraic object that happens to be a factorizable inverse monoid. This talk will be a gentle introduction to these reflection monoids.

Presentations for Temperley-Lieb algebras

James East
Western Sydney University

We give a new and conceptually straightforward proof of the well-known presentation for the Temperley-Lieb algebra, via an alternative new presentation. Our method involves twisted semigroup algebras, and makes use of two apparently new submonoids of the Temperley-Lieb monoid.

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