### Definition (Grothendieck et al., SGA1)

A *Galois category* consists of a pair $(\mathfrak{C}, F)$, where $\mathfrak{C}$ is an essentially small category

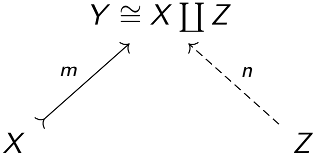
(GAL1) which is finitely complete,

(GAL2) has an initial object, finite coproducts, and quotients by finite groups of automorphisms,

(GAL3) every morphism $f : X \to Y$ factorizes as

$$\begin{array}{ccc} X & \xrightarrow{\quad f \quad} & Y \\ & {\scriptstyle e} \searrow \quad \nearrow {\scriptstyle m} & \\ & Z & \end{array}$$

where $m$ is a monomorphism and $e$ a strict epimorphism, (GAL4) for every monomorphism $m : X \to Y$, there exists a morphism $n : Z \to Y$



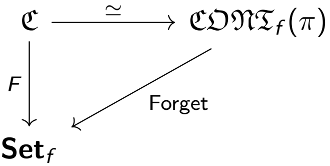

such that $(Y, m, n)$ becomes the coproduct of $X$ and $Z$,

together with a *fundamental functor* $F : \mathfrak{C} \to \mathbf{Set}_f$ to the category $\mathbf{Set}_f$ of finite sets, which

(GAL5) preserves the structures of (GAL1) and (GAL2), sends strict epimorphisms to surjections, and

(GAL6) is conservative (i.e. reflects isomorphisms: if $f$ is a morphism in $\mathfrak{C}$ such that $F(f)$ is an isomorphism, then $f$ is an isomorphism.)

Grothendieck's reconstruction theorem of SGA1 proves that a Galois category $(\mathfrak{C}, F)$ can be recovered by its fundamental functor under the equivalence




where $\pi = \mathrm{Aut}(F)$, topologized as a closed subgroup of $\prod_{A \in \mathfrak{C}} \mathrm{Aut}(F(A))$, is a Stone topological group.

### Theorem (Grothendieck et al.)

*The assignment* $(\mathfrak{C}, F) \mapsto \mathrm{Aut}(F)$, *induces an equivalence of categories* $\mathfrak{AUT} : \mathbf{GrothGal} \xrightarrow{\simeq_1} \mathbf{StoneGrp}$.

**Definition (Saavedra-Rivano, Deligne)**

A *(neutral) Tannakian category over a field k* consists of a pair $(\mathbb{C}, \omega)$, where $\mathbb{C}$ is a small category

(TAN1) which is symmetric monoidal category with tensor product $\otimes$ and tensor unit $\mathbf{1}$,

(TAN2) rigid (i.e. every object has a dual),

(TAN3) the endomorphism ring of the tensor unit satisfies $\mathrm{End}(\mathbf{1}) \cong k$,

(TAN4) and $\mathbb{C}$ is *k-linear abelian (as a symmetric monoidal category)*,

together with a monoidal *fiber functor* $\omega : \mathbb{C} \to \mathbf{Vec}_k^{\mathrm{f.d}}$ to the category of finite-dimensional $k$-vector spaces (endowed with the usual monoidal structure),

(TAN5) which is exact,

(TAN6) and faithful.

Deligne and Saavedra-Rivano's reconstruction theorem proves that $(\mathbb{C}, \omega)$ can be recovered by the (tensor) equivalence

$$\begin{array}{ccc} \mathbb{C} & \xrightarrow{\ \simeq_{\otimes}\ } & \mathbb{REP}^{\mathsf{f.d}}_{k}(\mathbb{G}) \\ {\scriptstyle \omega}\Big\downarrow & \swarrow {\scriptstyle \mathsf{Forget}} & \\ \mathbf{Vec}^{\mathsf{f.d}}_{k} & & \end{array}$$

### Theorem (Deligne and Saavedra-Rivano)

*The assignment (on* 0*-cells)*

$$(\mathbb{C}, \omega) \mapsto \mathrm{Aut}^{\otimes}(\omega),$$

*which associates to* $\omega$ *the affine* $k$*-group* $\mathrm{Aut}^{\otimes}(\omega) : \mathbf{CAlg}_k \to \mathbf{Grp}$ *that sends* $A$ *to the group of* $A$*-linear* $\otimes$*-automorphisms* $\omega(-) \otimes A$*, induces a biequivalence of* 2*-categories* $\mathbf{NTan}_k \xrightarrow{\simeq_2} \mathbf{AffGrpSch}_k$*.*