MT1642: EXAMPLE SHEET[1] 4

1.) Find the general solutions to the differential equations

(i) $\ddot{x} - \dot{x} - 2x = 40\cos(2t)$

(ii) $3y'' + 6y' + 6y = 30\sin(3x)$.

2.) For the same differential equations as in question 1, find the solutions that satisfy, respectively, the initial conditions

(i) $x = -6$ and $\dot{x} = -1$ when $t = 0$.

(ii) $y = -12/17$ and $y' = 0$ when $x = 0$.

3.) The motion of a mass-spring system is governed by the ODE

$$\ddot{x} + 4\dot{x} + 13x = 40\cos(3t).$$

Find the amplitude of the forced oscillations.

4.) Find the transient solutions of the following ODEs

(i) $y'' + y' + y = e^x$ (ii) $\ddot{x} + x = 1$

(iii) $\ddot{x} + x = t^2$ (iv) $\ddot{x} + x = \cos(t)$ [harder]

5.) Fig. 1 shows a diagrammatic sketch of the mechanical scales which the post office uses to weigh parcels. The weight of the parcel compresses the spring and allows its weight to be read off the scale on the right. A badly behaved customer might throw the parcel onto the initially unloaded scales. If there was no damping in the system, the parcel would then forever oscillate up and down and it would be impossible to determine its weight. Therefore, the ideal design of the scales incorporates a damper which is just strong enough to suppress oscillations for the maximum weight for which the scales have been designed. Assuming that the scales are supposed to work for parcels of mass $m < M$ and given that the spring constant is c, determine the minimum value of the damping constant k which ensures that oscillations are suppressed in the entire operational range of the scales.

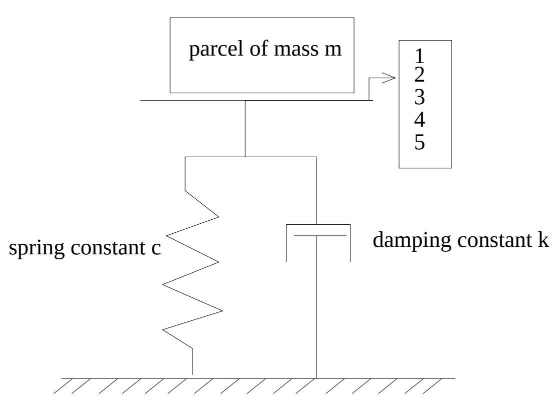

Figure 1: Sketch of mechanical scales.

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