A depth-averaged theory for large particle segregation, transport and accumulation in granular free-surface flows

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THE accumulation of larger less mobile particles at avalanche fronts can have a profound influence on the bulk motion of hazardous geophysical mass flows. The photograph in figure 1 shows an example of spontaneous self channelization, which occurred as a snow avalanche uprooted 200 m^2 of forest in the Puschlav valley. Such deposits are formed as larger less mobile material is segregated to the surface of the flow, where the velocity is greatest, and preferentially transported towards the flow front. Once it reaches the front, the large material can either be recirculated by particle size segregation, or it can be shouldered aside by the more mobile interior, to create stationary lateral levees that channelize the flow and enhance the overall run-out distance. This poster describes a new depth averaged theory for segregation that can easily be incorporated into the existing structure of typical geophysical mass flow models (e.g. Savage & Hutter 1989, Gray *et al.* 2003) and opens up the realistic possibility of describing some of these complex processes in detail.

Figure 1: Snow avalanche with levees formed from debris at Val Prada, Switzerland (Courtesy P. Bartelt, WSL-Institut für Schnee- und Lawinenforschung, Davos).

Derivation of a depth-averaged segregation model

Granular avalanches are very effective at sorting particles by size, through a combination of *kinetic sieving* and *squeeze expulsion* (Savage & Lun 1988). Considerable progress has been made on modelling these effects (see e.g. Gray & Thornton 2005, Thornton *et al.* 2006, Gray & Chugunov 2006) and these theories take the form

$$\frac{\partial\phi}{\partial t}+\frac{\partial}{\partial x}(\phi u)+\frac{\partial}{\partial z}(\phi w)-\frac{\partial}{\partial z}\Big(S_r\phi(1-\phi)\Big)=\frac{\partial}{\partial z}\left(D_r\frac{\partial\phi}{\partial z}\right), \tag{1}$$

where $0\le\phi\le 1$ is the volume fraction of small particles per unit granular volume, S_r is non-dimensional segregation rate, D_r is the coefficient of diffusive remixing and (u,w) are the velocity components in the downslope and normal directions (x,z). The volume fraction of large particles is equal to $1-\phi$. Defining the depth-averaged concentration of small particles and the depth averaged flux of small particles as

$$\bar\phi=\frac{1}{h}\int_b^s\phi\,dz \quad \text{and} \quad \overline{\phi u}=\frac{1}{h}\int_b^s\phi u\,dz, \tag{2}$$

the segregation-remixing equation (1) can be integrated through the avalanche depth, subject to no flux and kinematic conditions at the surface and base of the avalanche, to give

$$\frac{\partial}{\partial t}(h\bar\phi)+\frac{\partial}{\partial x}(h\overline{\phi u})=0. \tag{3}$$

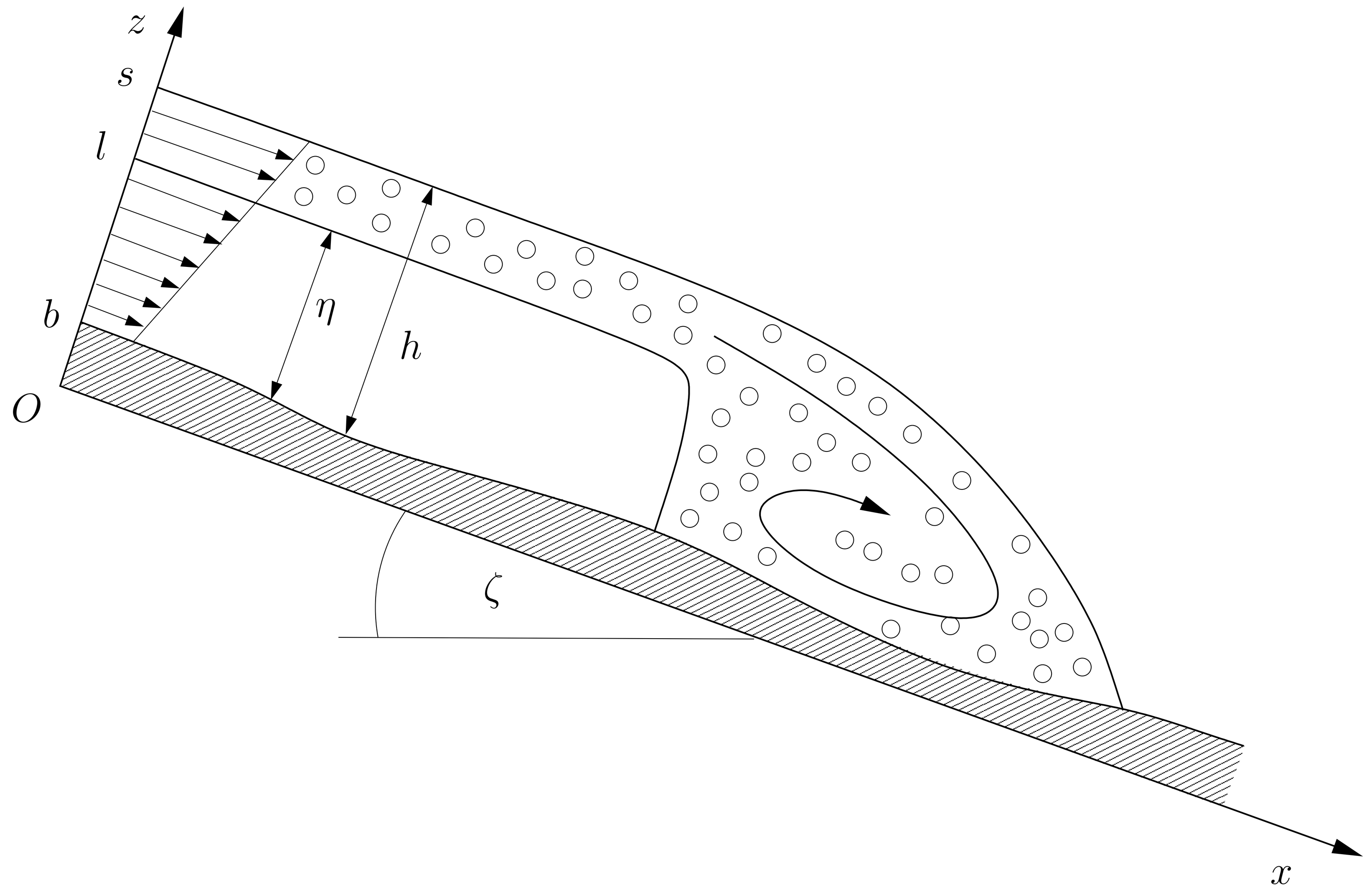

Figure 2: A schematic diagram of an avalanche flow front and the inversely graded particle size distribution within the interior. Large particles rise up into the faster moving near surface layers of the avalanche and are transported to the flow front, where they can be over-run and then recirculated, by particle size segregation, to form a bouldery margin.

To close the model expressions must be devised for $\bar\phi$ and $\overline{\phi u}$ defined in (2). The stratification pattern experiments of Gray & Ancey (2009) showed that:-

- inverse grading could develop very rapidly,
- grains could be very sharply segregated by size, and
- there was strong shear through the avalanche depth.

We therefore assume that there is a linear velocity profile with depth and the particle size distribution is always sharply inversely graded

$$u=\alpha\bar u+2(1-\alpha)\bar u\left(\frac{z-b}{h}\right), \qquad \phi=\begin{cases}0, & l<z\le s,\\ 1, & b\le z\le l.\end{cases} \tag{4}$$

which implies that

$$h\bar\phi=\eta, \qquad h\overline{\phi u}=\eta\bar u-(1-\alpha)\bar u\eta\left(1-\frac{\eta}{h}\right). \tag{5}$$

Substituting (5) into (3) yields an important new large particle transport equation for the evolution of the inversely graded shock interface height η

$$\frac{\partial\eta}{\partial t}+\frac{\partial}{\partial x}(\eta\bar u)-\frac{\partial}{\partial x}\left((1-\alpha)\bar u\eta\left(1-\frac{\eta}{h}\right)\right)=0. \tag{6}$$

Using $\eta=h\bar\phi$ this can also be rewritten as an equation for the depth-averaged concentration of small particles

$$\frac{\partial}{\partial t}(h\bar\phi)+\frac{\partial}{\partial x}(h\bar\phi\bar u)-\frac{\partial}{\partial x}\Big((1-\alpha)h\bar u\bar\phi\left(1-\bar\phi\right)\Big)=0. \tag{7}$$

Remarkably this is very closely related to the segregation equation (1) from which it is derived. While the large particle transport equation has a very simple representation of the particle size distribution, it does a surprisingly good job of capturing solutions to the full theory once the grains have segregated into inversely graded layers. In particular, provided the inversely graded interface does not break it has precisely the same solution as the full theory (Gray, Shearer & Thornton 2006). When the interface does break, a discontinuous jump forms instead of a breaking size segregation wave (Thornton & Gray 2008), but the net transport of large particles towards the flow front is exactly the same. Figure 3 shows an exact solution that illustrates how large particles are transported to the flow front and then accumulate there by instantaneous recirculation at a shock in the interface height. The new transport equation (6) opens up the possibility of coupling the evolving particle size distribution to existing depth averaged models for granular avalanches (e.g. Savage & Hutter 1989, Gray *et al.* 2003) to study segregation-mobility feedback effects that lead to levee formation and fingering (e.g. Pouliquen & Vallance 2001).

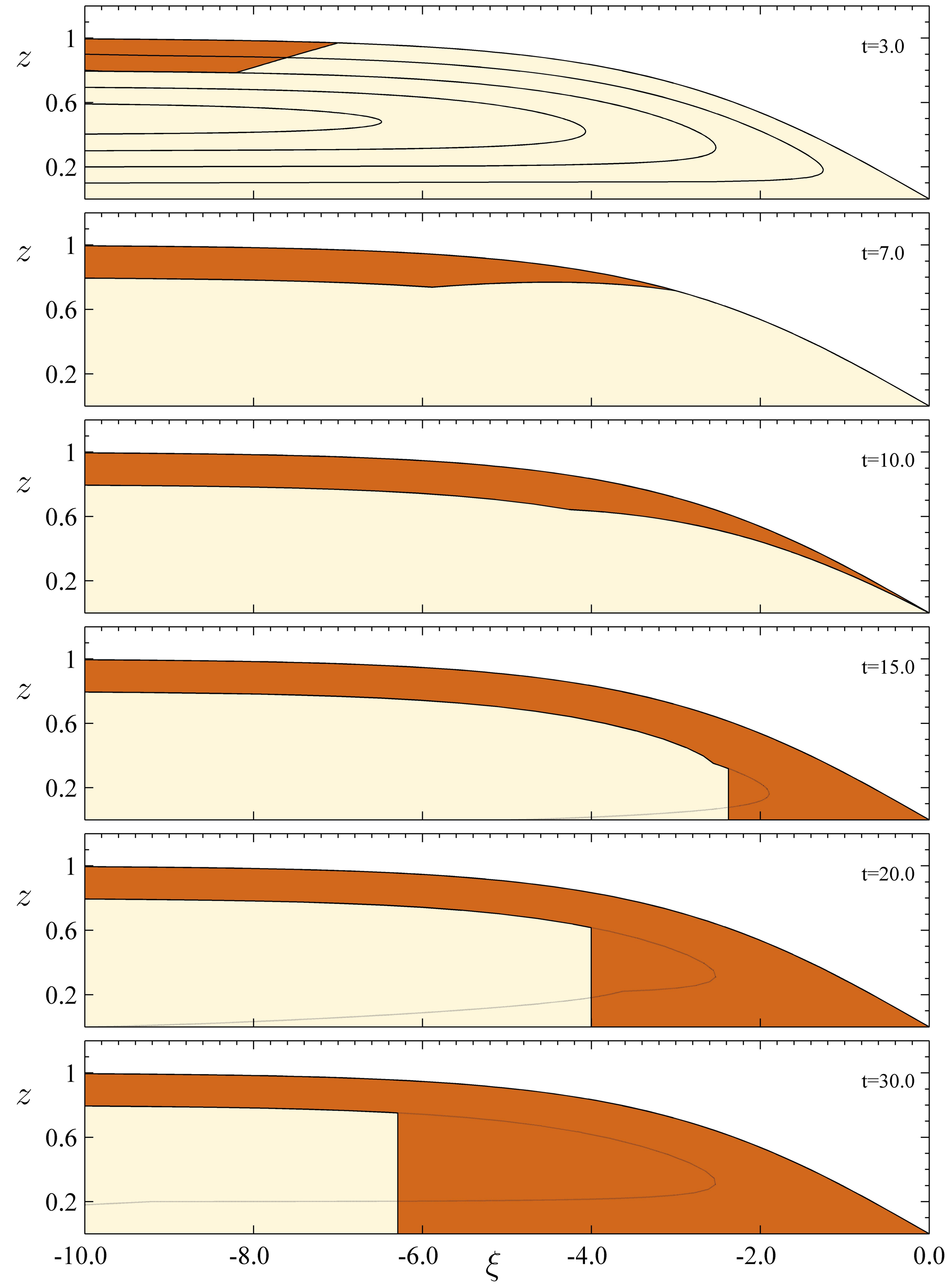

Figure 3: A series of vertical sections ($\xi=x-u_Ft, z$) through an avalanche front propagating at constant speed u_F. At $t=0$, the front is entirely composed of small (beige) particles. As time proceeds large (brown) particles are advected towards the flow front, where they start to accumulate after $t=10$. The inversely graded avalanche is connected to the large particle rich front by a discontinuity in the interface height, whose position is found by a shock fitting procedure.

- Large particle segregation, transport and accumulation in granular free-surface flows. Gray J.M.N.T. & B.P. Kokelaar (2010) J. Fluid Mech. **652**, 105–137.
- Segregation, recirculation and deposition of coarse particles near two-dimensional avalanche fronts. Gray J.M.N.T. & C. Ancey (2009) J. Fluid Mech. **629**, 387–423.
- Breaking size-segregation waves and particle recirculation in granular avalanches. Thornton A.R. & J.M.N.T. Gray (2008) J. Fluid Mech. **596**, 261–284.
- Particle-size segregation and diffusive remixing in shallow granular avalanches. Gray J.M.N.T. & V.A. Chugunov (2006) avalanches. J. Fluid Mech. **569**, 365–398.
- Time-dependent solutions for particle size segregation in shallow granular avalanches. Gray J.M.M.T., Shearer M. & A.R. Thornton. (2006) Proc. Roy. Soc. **462**, 947–972.
- A three-phase mixture theory for particle size segregation in shallow granular free-surface flows. Thornton A.R., Gray J.M.N.T. & A.J. Hogg. (2006) J. Fluid Mech. **550**, 1–25.
- A theory for particle size segregation in shallow granular free-surface flows. Gray J.M.N.T. & A.R. Thornton (2005) Proc. Roy. Soc. A **461**, 1447–1473.
- Shock waves, dead-zones and particle-free regions in rapid granular free-surface flows. Gray J.M.N.T., Tai Y.C. & S. Noelle (2003) J. Fluid Mech. **491**, 161–181.
- Segregation induced instabilities of granular fronts. Pouliquen O. & J.W. Vallance (1999) Chaos **9**(3), 621–630.
- The motion of a finite mass of granular material down a rough incline. Savage S.B. & K. Hutter (1989) J. Fluid Mech. **199**, 177–215.
- Particle size segregation in inclined chute flow of dry cohesionless granular solids. Savage S.B & C.K.K. Lun (1988) J. Fluid Mech. **189**, 311–335.