PARTICLE SIZE SEGREGATION, GRANULAR SHOCKS AND STRATIFICATION PATTERNS

J.M.N.T. GRAY AND Y.C. TAI
Institut für Mechanik,
Technische Universität Darmstadt,
64289 Darmstadt, Germany.

1. Stratification Patterns

Large scale stratification patterns [1] are formed when a mixture of two grain sizes, or more, repeatedly avalanche downslope and are brought to rest by upslope shock wave propagation. The avalanches are generated by either surface deposition, basal erosion or rotation of the free surface. Provided each of these processes take place at sufficiently low rates the avalanches occur intermittently, due to the difference between the static and dynamic friction angles [2], [3]. Segregation of the particles takes place within the flowing avalanche by a process called *kinetic sieving* [4], [5]. An initially homogeneous mixture of grains is rapidly transformed into an *inversely graded* particle size distribution, in which the large particles overlie the smaller ones. The reason for this segregation is simple. As grains are sheared within the avalanche void spaces are continually being created and annihilated below each grain, and the smaller grains are more likely to fall into the available space than the large grains. For a bi-disperse granular material two segregated layers are rapidly generated within the avalanche, as shown in the photograph and schematic diagram in Fig. 1. An additional velocity shear through the depth of the avalanche transports the larger particles to the front and the smaller ones to the rear. This can also be seen in Fig. 1.

When the front of the avalanche comes to rest on a run-out plane, or encounters an obstacle, a shock wave can be generated, which propagates rapidly upslope *freezing* the segregation pattern into the deposited material. These granular shocks are travelling waves that are similar to hydraulic jumps or bores in fluid dynamics. For a bi-disperse granular avalanche the

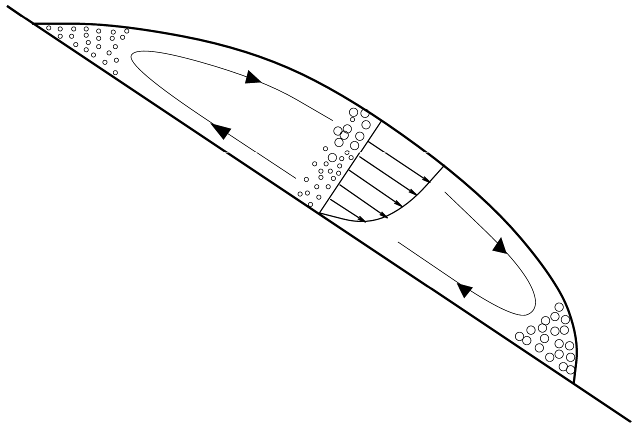

Figure 1. The photograph shows a flowing avalanche at the two uppermost layers of a stratified stationary deposit of a bi-disperse granular material. Kinetic sieving produces an inversely graded particle size distribution within the avalanche, in which the larger (white) particles overlie the smaller (dark) ones to produce a two layer shear band. Additional velocity shear through the avalanche depth causes the larger particles to migrate towards the front and the smaller ones to the rear, as shown in the schematic diagram.

two segregated layers thicken as the shock passes through them to leave two strata, or a *stripe*, at the current free surface (Fig. 2), which is later buried. Successive repetition of these processes builds up a large scale stratification pattern that strongly reflects the avalanche history and dynamics. For instance, in the patterns shown in Figs. 1 & 2 each pair of strata corresponds to the passage of a single avalanche. Stripes are therefore the basic building blocks of large scale pattern formation.

2. A Simple Model For Stripe Formation

The one-dimensional Savage–Hutter theory [6], [7] has been used extensively to model the flow of granular avalanches, and it forms the basis of the simple model described here. A slope fitted curvilinear coordinate system is adopted in which the z axis is normal to, and the x axis parallel to, the local slope topography. The $z = 0$ plane therefore coincides with the topography of the chute. The shallowness of the avalanche is exploited to integrate the mass balance equation through the avalanche thickness. Using the kinematic boundary conditions on the free surface and base a continuity equation is derived for the avalanche thickness h

$$h_t + (uh)_x = 0, \tag{1}$$

where the subscripts t and x indicate differentiation with respect to time and the downslope coordinate, and u is the depth averaged downslope ve-

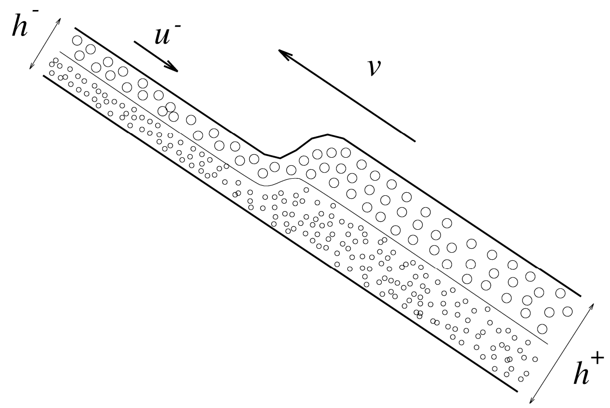

Figure 2. The photograph shows the propagation of a granular shock on the free surface of a stratified stationary deposit. The flow in the two layer shear band is brought to rest, as the jump propagates upslope, freezing the particle size distribution into the material to form a stripe.

locity. An analogous depth integration of the downslope momentum balance yields an equation for the velocity

$$(hu)_t + (hu^2 + \cos\zeta K h^2/2)_x = h\cos\zeta(\tan\zeta - (u/|u|)\tan\delta), \qquad (2)$$

where $\zeta = \zeta(x)$ is the slope inclination and δ is the basal angle of friction. For simplicity additional curvature effects have been neglected and the equations are presented in conservative form. The earth pressure coefficient K determines the ratio of in-plane to normal pressure [6] and assumes two values, *active* and *passive*, dependent on whether the downslope motion is divergent or convergent. These are given by

$$K_{\text{act/pas}} = 2\sec^2\phi\left(1 \mp \sqrt{1-\cos^2\phi\sec^2\delta}\right) - 1, \qquad (3)$$

where ϕ is the internal angle of friction.

It is assumed here that a particle size distribution exists within the avalanche, but that the motion of the bulk material is not influenced by the local volume fraction of the respective sizes. It is also clear from the experiments that the kinetic sieving time scale is much faster than that of the bulk avalanche motion. The simplest model of stripe formation is therefore to assume that the particles are *presorted* when they are input into the flow, with the large particles on top and the small particles below. The volume fraction of small particles per unit mixture volume, ν, is therefore simply advected with the flow. That is $0 \le \nu \le 1$ acts as a tracer

$$\nu_t + u\nu_x + w\nu_z = 0. \qquad (4)$$

The vertical velocity w is computed from the incompressibility condition $u_x + w_z = 0$, subject to the boundary condition that $w = 0$ at $z = 0$. Assuming that the downslope velocity is independent of the avalanche depth it follows that the vertical velocity

$$w(z) = -zu_x, \tag{5}$$

at height z above the chute. Equations 4 and 5 enable the presorted particle size distribution of the input material to be tracked within the flowing avalanche.

3. Numerical Simulations

The original Lagrangian numerical scheme [6] encounters problems when shocks occur within the avalanche. A fixed grid, shock capturing, TVD method [8] using the 'superbee' slope limiter has therefore been developed to solve the system of equations 1-5.

An initial chute profile is defined that consists of a plane inclined at $\zeta = 36°$ to the horizontal, which is connected to a horizontal run-out plane by a smooth transition zone. The avalanche is assumed to have basal angle of friction $\delta = 35°$ and internal angle of friction $\phi = 38°$. These material parameters and basal geometry approximate the conditions on the side of pre-existing pile of granular material. An avalanche is initiated at the top of the chute by prescribing a constant input rate of pre-sorted granular material, with the lower half occupied with small particles and the upper half filled with large particles.

The results of the simulation are shown in Fig. 3. The avalanche propagates downslope transporting the initial particle size distribution as a tracer quantity within the flow. This generates the two layer shear band similar to that in Fig. 1. When the avalanche reaches the run-out plane the gravity forcing ceases and a granular shock develops, which then propagates upslope with approximately constant speed. As the shock passes through a given point the avalanche thickens and the thickness of the inversely graded layers increases, in the same way as observed (Fig. 2), to form a stripe of stationary material.

4. Large Scale Patterns

Once one stripe has formed its free surface forms the new basal topography on which the *stripe formation mechanism* (i.e. an avalanche followed by a granular shock) can be repeated, and in this way a large scale stratification pattern is generated. For instance, the stratification patterns shown in Figs. 1& 2 were generated by constant deposition from a point source. Each

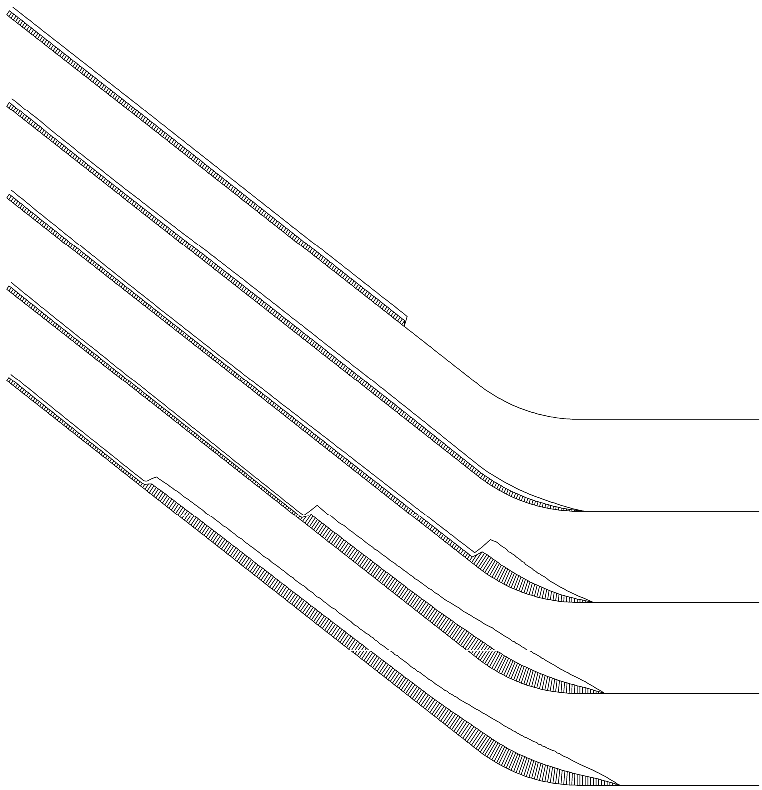

Figure 3. The computed formation of a single stripe is illustrated at a sequence of time steps. The shaded regions contain a high concentration of small particles and the unshaded regions have low ν. The avalanche flows down the inclined chute until the front reaches the horizontal run-out plane, where it comes to rest. A shock wave then develops in which the thinner flowing material on the inclined plane is rapidly brought to rest to form a thick stripe that preserves the inversely graded particle size distribution.

pair of layers corresponds to the passage of an avalanche and the transition from white to black particles, as one moves up through the deposit, marks the position of an earlier basal surface.

It is also possible to generate different stratification patterns by erosion at a point source [1] or by rotation. Figure 4 shows the stratification pattern that is generated when a thin disk, 3/4 filled with bi-disperse granular material, is rotated at constant rates of 110 and 20 seconds per revolution. At low revolution rates avalanches are released intermittently and are brought to rest by a shock wave initiated as the avalanche front reaches the side of the disk. A stripe is formed tangent to the free surface, which is then rotated and buried. After a complete revolution of the disk a Catherine wheel pattern is formed about the rigid central core. At faster rotation

Figure 4. At low revolution rates stratification patterns develop in a thin disk filled with bi-disperse granular material (left photo). At faster revolution rates a continuous flow regime develops, where the intermittency and shock waves are suppressed, and a different pattern is formed (right photo).

rates a continuous flow regime is entered in which the intermittency and shock waves disappear. The free surface is instead quasi-steady and there is a continuous particle size distribution outside the central core [9] with the larger particles concentrated near the edge of the disk.

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