

# Supplementary Information for: Transient wave activity in snow avalanches is controlled by entrainment and topography

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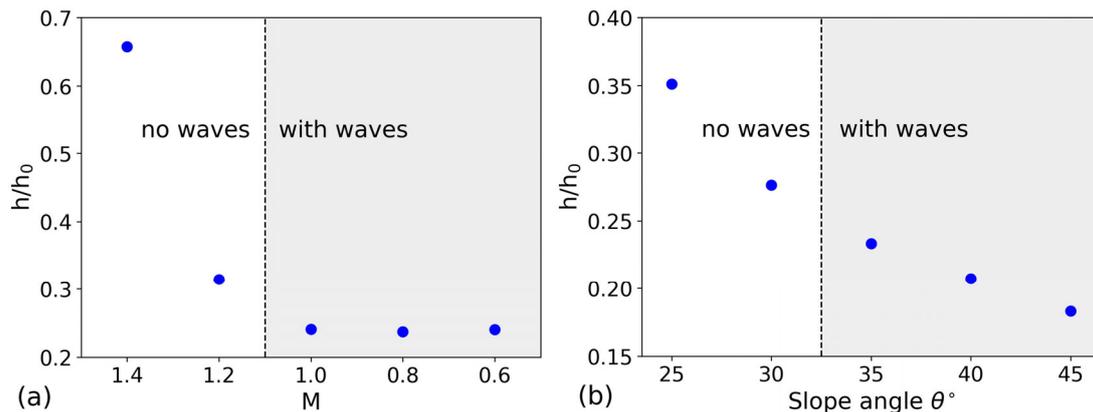
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## Supplementary Note 1 - Effect of snow friction and slope angle on the occurrence of waves

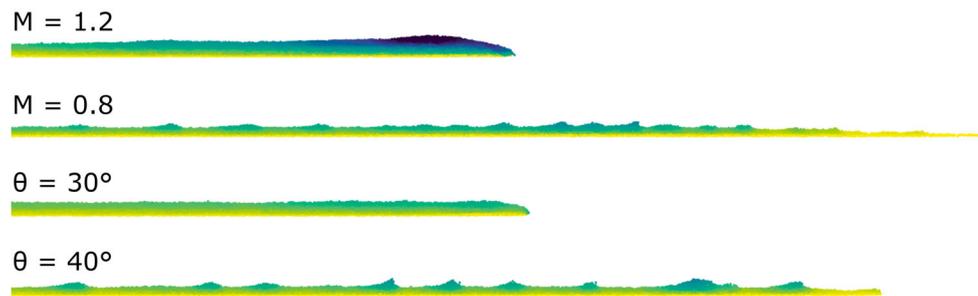
To investigate the effect of snow friction  $M$  and slope angle  $\theta$  on the occurrence of waves, two groups of simulations were conducted with other numerical parameters (e.g., mesh size, time step) fixed. The evolution of the height of deposited snow  $h$  from the simulations are shown in Supplementary Figure 1. It is observed from Supplementary Figure 1a that smaller snow friction  $M$  tends to give smaller deposit height  $h$  when  $M$  decreases from 1.4 to 1.0, and the tendency decreases with the reduction of  $M$  from 1.0 to 0.6. From Supplementary Figure 1b, the reduction of the slope angle  $\theta$  leads to the decrease of the deposit height  $h$ , which is consistent with the trend in Figure 2 in the manuscript.



Supplementary Figure 1: Evolution of the height of deposited snow  $h$  with (a) snow friction  $M$  and (b) slope angle  $\theta$ . The initial sample height  $h_0$  is 0.2 m

For the simulations cases in Supplementary Figure 1, waves are observed with small snow friction ( $M = 1.0, 0.8, 0.6$ ) and large slope angle ( $\theta = 35^\circ, 40^\circ, 45^\circ$ ). Typical cases with and without waves are presented in Supplementary Figure 2. It is found that the avalanches with waves have larger velocity and smaller flow height compared to

that without waves. Therefore, the occurrence of waves can be associated with large Froude number. This is consistent with existing studies on the identification of occurrence of waves in granular flows<sup>[1-2]</sup>. For example, it was concluded that a Froude number larger than  $2/3$  results in the occurrence of waves<sup>[2]</sup>. Please note that it is difficult to get one representative Froude number from each simulated avalanche in our preliminary study (Supplementary Figure 1), because the avalanches are not steady and their Froude numbers change with time. Nevertheless, according to the two groups of simulations, we prove that the occurrence of waves in the simulated avalanche is due to physical reasons instead of numerical instabilities.



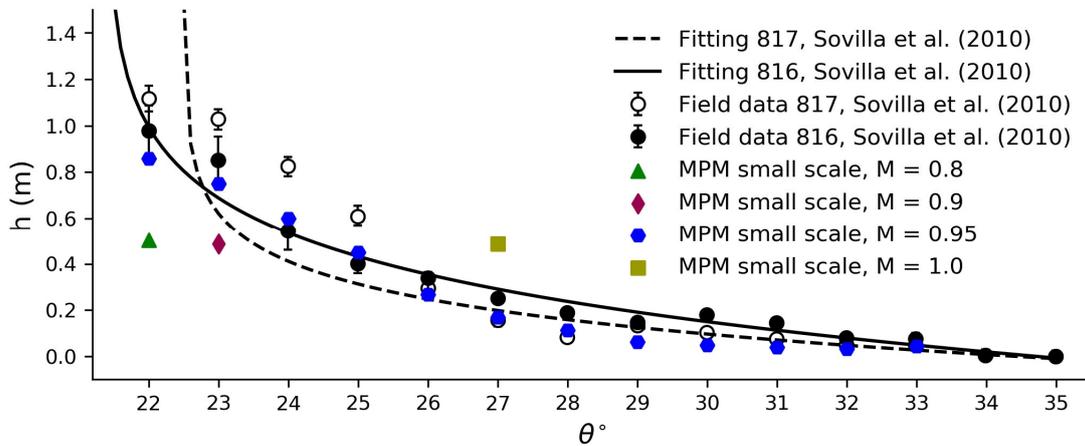
Supplementary Figure 2: Effect of snow friction  $M$  and slope angle  $\theta$  on the occurrence of waves

## Supplementary Note 2 - Sensitivity of model parameters

To clarify the influence of model parameters on the simulation results, key parameters, including snow properties and terrain features, have been changed to obtain complementary information for better explanation of the simulation results in the manuscript.

The friction coefficient of snow  $M$  is a factor which was calibrated to be 0.95. Four cases with  $M$  from 0.8 to 1.0 were firstly conducted with other parameters fixed, and the deposit height was plotted with the slope angle. In these four cases, the slope angles at which the avalanches deposit are respectively  $22^\circ$ ,  $23^\circ$ ,  $25^\circ$ ,  $27^\circ$  as shown in Supplementary Figure 3. It is found that  $M = 0.8$  and  $M = 0.9$  lead to underestimated deposit height  $h$ , while  $M = 1.0$  gives overestimated  $h$ . The agreement between the simulation result with  $M = 0.95$  and the field data<sup>[3]</sup> is satisfactory. As shown in the figure, the deposit height  $h$  in the four simulation cases is close (around 0.5 m), while the slope angle  $\theta$  corresponding to  $h$  differs obviously. The smaller the snow friction  $M$ , the lower the slope angle  $\theta$ . This is reasonable since a small friction  $M$  gives more mobility of a granular flow and thus more difficulty for the flow to stop on a steep slope.

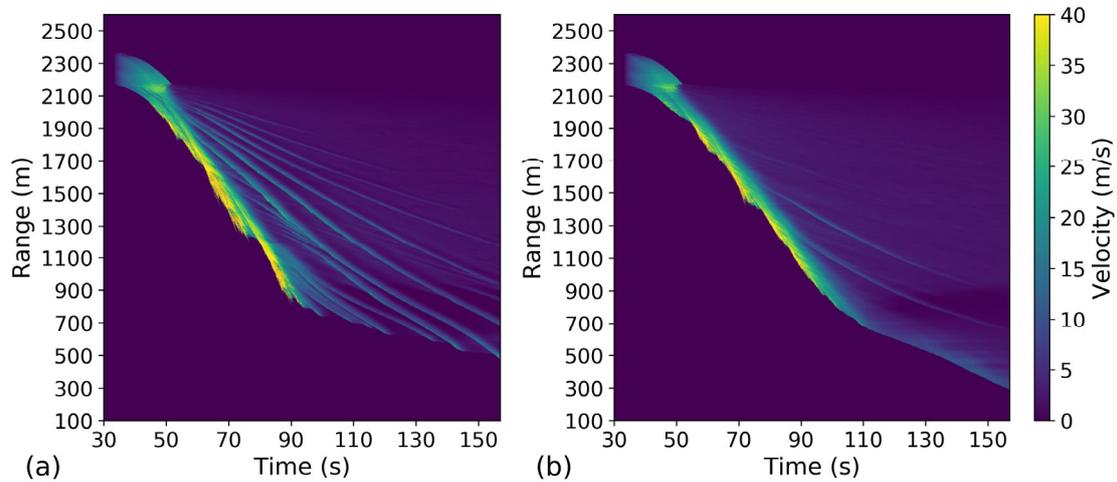
In these four cases, the initial height of the released snow sample  $h_0$  is 0.6 m. As  $M = 0.95$  gives good consistency with the field data, it is further used to obtain the deposit height at other slope angles in Supplementary Figure 3. By varying  $h_0$  (as summarized in Table 2 in the manuscript) and slope angle  $\theta$  while fixing other parameters (e.g.,  $M = 0.95$ ), the deposit height at other slope angles can be obtained. As observed in Supplementary Figure 3,  $M = 0.95$  gives good consistency with the field data at different slope angles. Therefore,  $M = 0.95$  was used in the simulation of the VdIS avalanche.



Supplementary Figure 3: Evolution of the height of deposited snow  $h$  with the slope angle  $\theta$

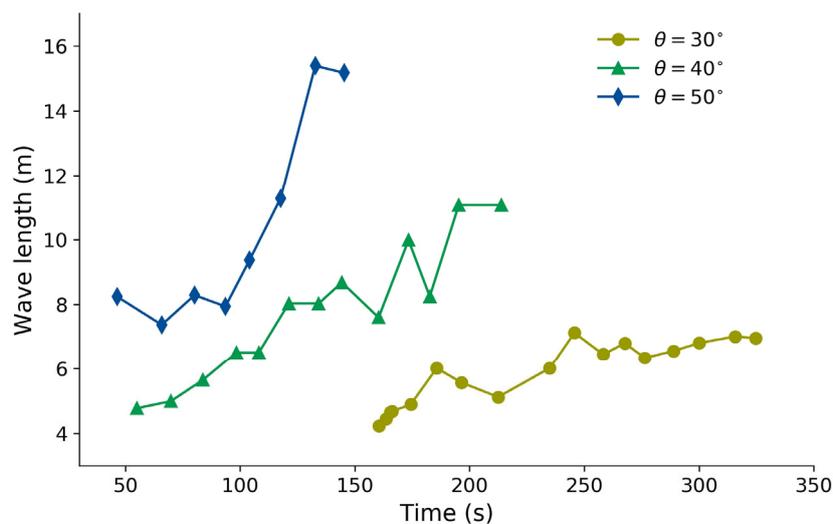
To investigate the effect of snow properties on the wave behavior considering the complex VdIS terrain, two additional simulations were conducted and the spatial-temporal evolution of the avalanche behavior was exported. As shown in Supplementary Figure 4, by changing the snow properties, the wave behavior (e.g., wave number, wave velocity, wave transition) changes. Therefore, it is clear that the snow properties can be changed to match the avalanche behavior observed in the field. However, to avoid over-parameterization, we have calibrated the snow properties

before applying them to the simulation of the VdIS avalanche. Please note that the two cases in Supplementary Figure 4 are preliminary results showing the effect of snow properties on wave behavior, more constructive and systematic simulations need to be conducted in the future to delicately explore the influence of each snow parameter.



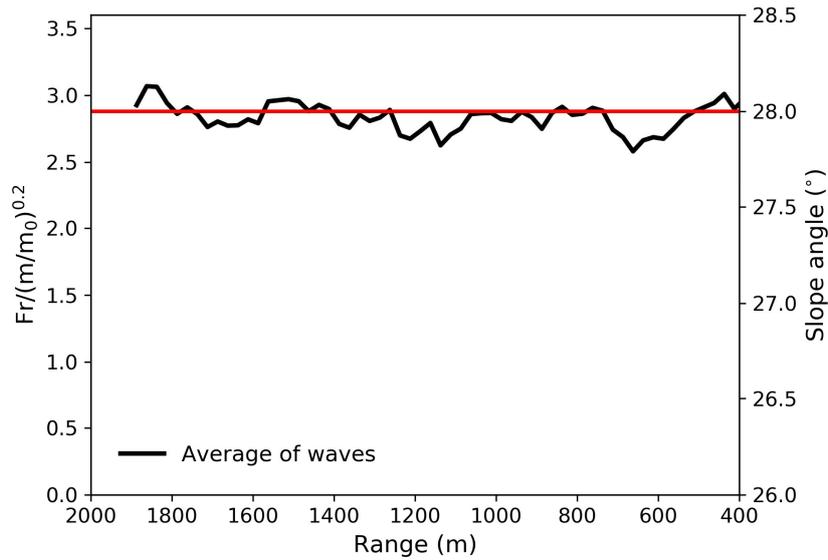
Supplementary Figure 4: Effect of snow properties on the evolution of wave behavior. (a)  $\rho = 250 \text{ kg/m}^3, M = 0.5, \beta = 0, \zeta = 1, p_0^{ini} = 3 \text{ kPa}, E = 3 \text{ MPa}$ ; (b)  $\rho = 150 \text{ kg/m}^3, M = 0.5, \beta = 0, \zeta = 0.5, p_0^{ini} = 40 \text{ kPa}, E = 1 \text{ MPa}$

According to the conclusion from the manuscript, the terrain has a strong correlation with the wave behavior. A pilot study on the effect of slope angle on the wave features under idealized conditions was conducted. As shown in Supplementary Figure 5, the steeper the slope angle, the longer the length of the waves in the avalanche. Hence, there is indeed a strong correlation between the terrain feature and the wave behavior. It should be noted that, in addition to slope angle, other terrain features, such as curvature and roughness, could also affect the wave behavior, and need to be explored in the future.



Supplementary Figure 5: Evolution of wave length with time considering different slope angles

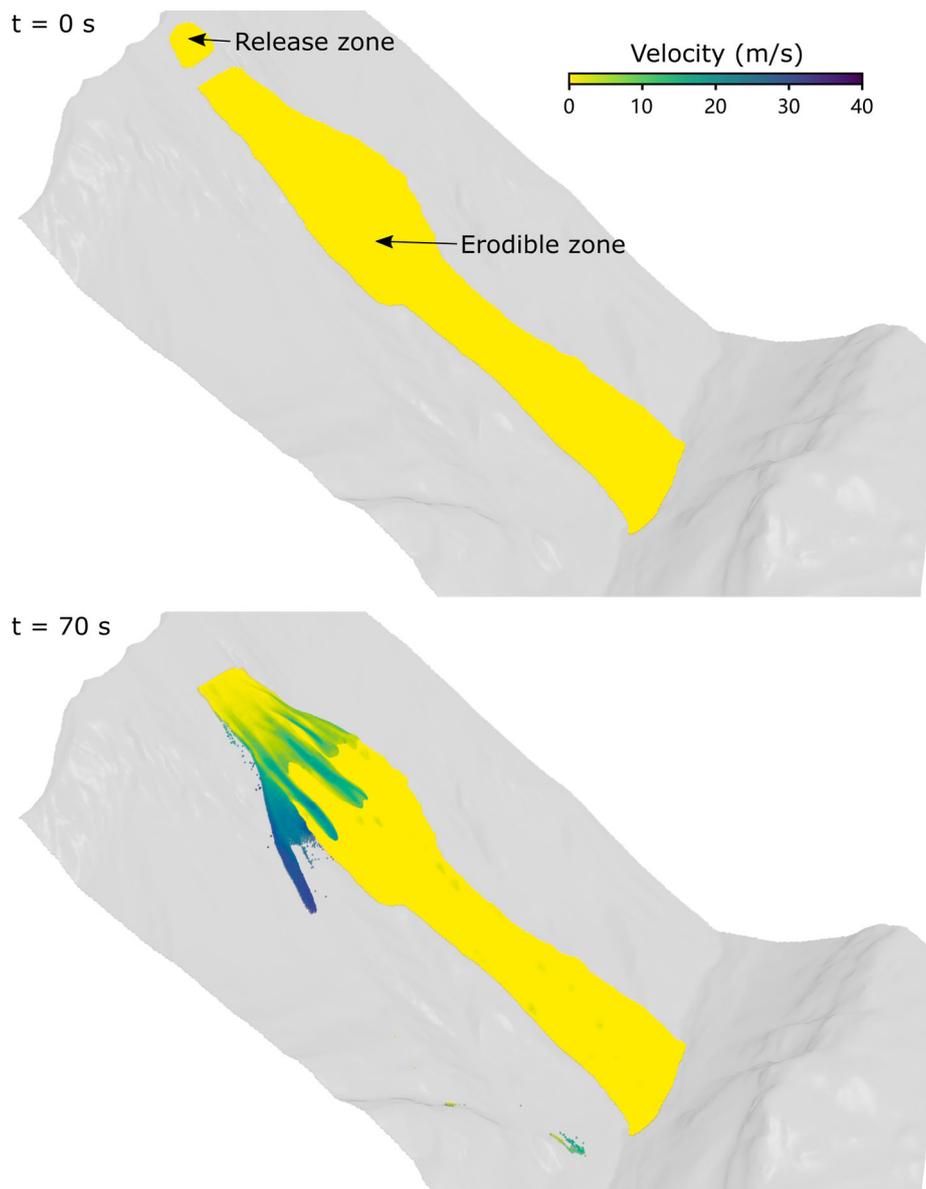
The investigation on the average normalized Froude number shows that the effect of the slope angle of the terrain on the dynamic wave behavior is strong. To further verify the correlation between the average normalized Froude number and the slope angle (Figure 5d in the manuscript), an avalanche on a plane with a constant slope angle of  $28^\circ$ , which is the average slope angle of the VdIS terrain, was simulated. As shown in Supplementary Figure 6, the average normalized Froude number of the avalanche waves fluctuated around a constant value, showing the governing effect of the constant slope angle.

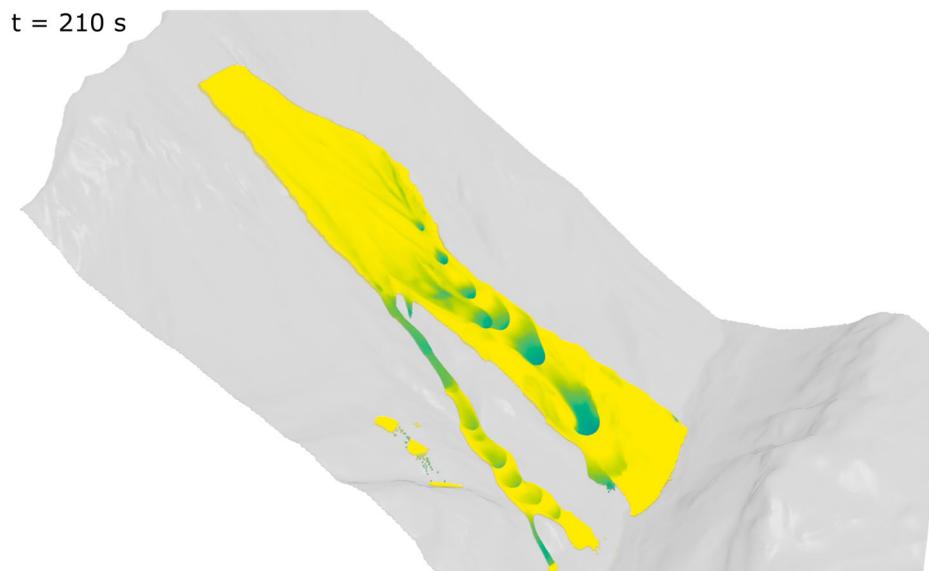
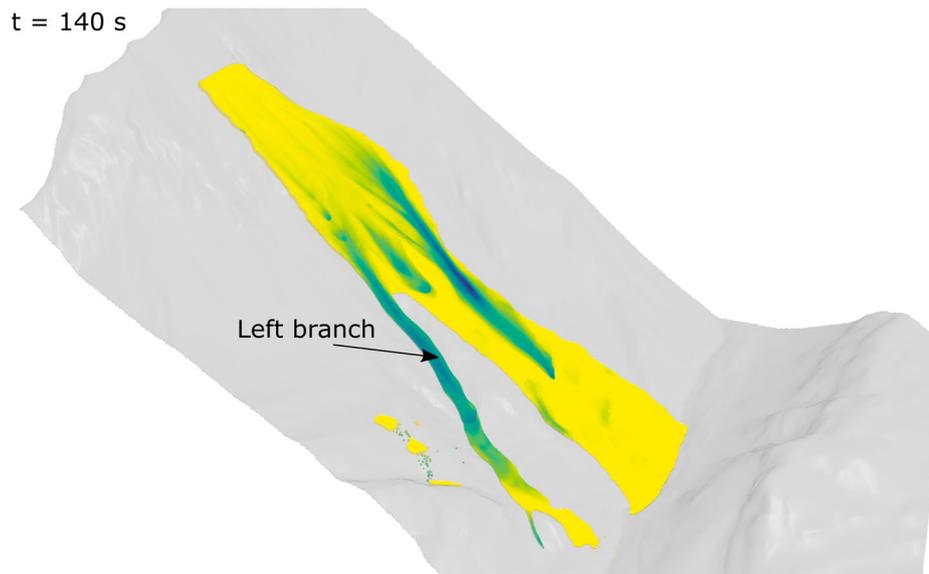


Supplementary Figure 6: Evolution of the average normalized Froude number of all the waves and the slope angle of a plane with constant inclination of  $28^\circ$

### Supplementary Note 3 - Three-dimensional simulation of the Vallée de la Sionne avalanche

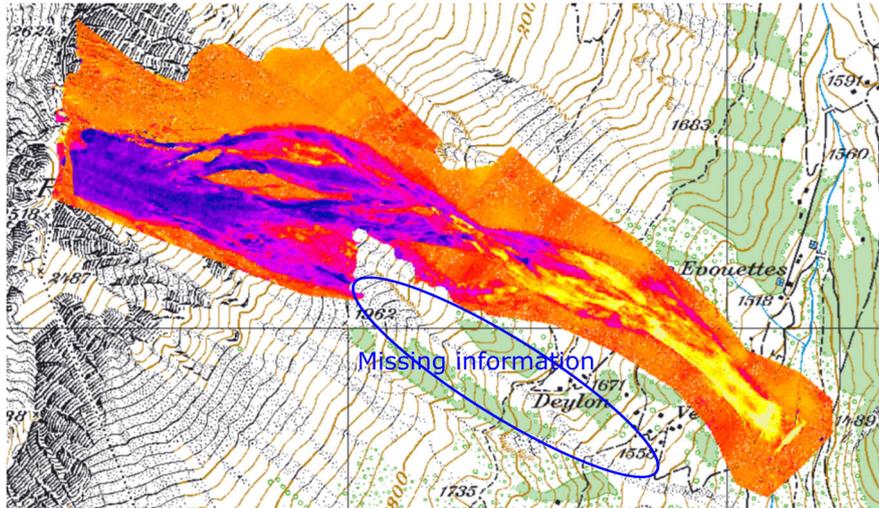
The simulation of the avalanche at Vallée de la Sionne (VdS) analyzed in the manuscript was conducted with two-dimensional (2D) material point method (MPM), as it is more computationally efficient and easier to set boundary conditions compared with three-dimensional (3D) modeling. To confirm that the 2D simulation can capture key wave features of the VdS avalanche, a preliminary 3D simulation has been conducted with assumptions. As shown in Supplementary Figure 7, at the initial state ( $t = 0$  s), a release zone is assumed upstream of an erodible zone, which is placed along the major flow path of the avalanche. Fingering behavior is observed as the avalanche goes to the erodible zone ( $t = 70$  s). When the avalanche reaches the downstream of the slope (from  $t = 140$  s to  $t = 210$  s), there are a series of erosion-deposition waves appeared in the avalanche. These wave features are similar as observed in the 2D simulation. Therefore, the 2D simulation is efficient in capturing the key wave features of the VdS avalanche.





Supplementary Figure 7: The simulated VdIS avalanche with 3D MPM

It should be noted that the field investigation on this avalanche mainly focused on the right branch of the avalanche, while some data on the left branch is missing (see Supplementary Figure 8). The boundary of the erodible zone in the 3D simulation leads to artificial uncertainty of the simulation result. Thus, it is difficult to run a complete 3D simulation and do one-to-one comparison of the simulation result with the field data. In the future, the 3D simulation needs to be improved with field information on the boundary conditions, and with optimized computational algorithms for less computational cost.



Supplementary Figure 8: Missing information of the left branch of the avalanche from the field investigation using laser scanning

### **Supplementary References**

- [1] Balmforth, N. J., & Mandre, S. (2004). Dynamics of roll waves. *Journal of Fluid Mechanics*, 514, 1-33.
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- [3] Sovilla, B., McElwaine, J. N., Schaer, M., & Vallet, J. (2010). Variation of deposition depth with slope angle in snow avalanches: Measurements from Vallée de la Sionne. *Journal of Geophysical Research: Earth Surface*, 115(F2).