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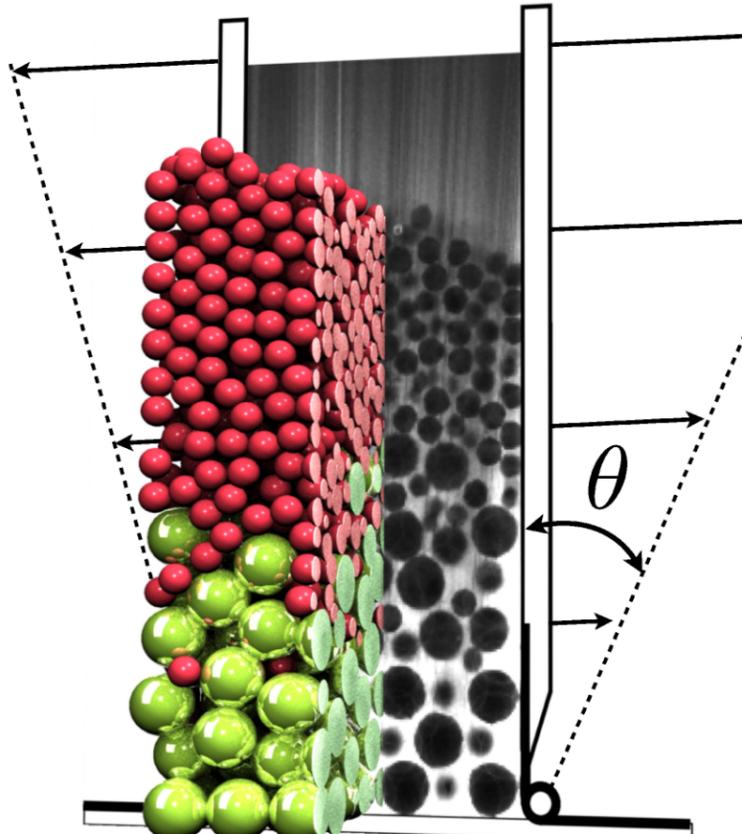


Figure 1: Shearbox set-up showing motion of sidewalls. Black and white image is a raw RIMS image, and colour image is a visualisation of particle positions.

Introduction

When subjected to agitation or shear, granular materials display a natural tendency to self-organise, with different sized constituents counter-intuitively separating from each other. Not only does this segregation produce complex and beautiful patterns, but it also provides serious challenges to both industrial and geophysical environments. A non-homogeneous mixing can critically degrade the product quality, whilst segregation can form levees that increase the flow mobility and run-out. Yet, despite its importance, understanding individual particle motion during segregation has been problematic with conventional techniques such as binning and sidewall observation. Here, a classical shearbox with the novel 'Refractive Index Matched Scanning' (RIMS) technique has been used to give new physical insights into segregation.

Methods

A shearbox 51mm deep and 37mm wide was filled to a height of 87 ± 3 mm with a bidisperse mixture of borosilicate glass spheres of diameters $d_l=8$ mm and $d_s=4$ mm. The larger spheres were placed at the bottom, the surface flattened and the smaller particles on top. The sidewalls oscillate to a maximum angle $\theta_{max}=30^\circ$, while remaining parallel, applying a periodic shear with period $T=13$ s. The mixture was analysed using RIMS: a mixture of benzylalcohol and ethanol with the same refractive index as the glass spheres was used for the interstitial fluid, together with a fluorescent dye. A 532nm laser sheet illuminated the mixture perpendicular to the oscillating walls, giving a stack of vertical cross sections with the particles appearing as dark circles, as shown in figure 1. Scans were performed after each full oscillation, with the shearbox in the upright position. The stack of images were processed using convolution to give three dimensional particle positions, with the temporal particle evolution in figure 3 uncovering an underlying asymmetry in the large and small particle behaviour.

Asymmetric Behaviour of the Two Species

Small particles percolate downwards and collect at the bottom, whilst large particles rise upwards and accumulate at the surface, as shown in figure 3. It is observed that large particles rise slowly upwards through regions of many small particles, whilst all the small particles percolate down quickly. This distinct asymmetric behaviour between the two species was measured on both bulk and particle scales. The asymmetry can be modelled using a cubic flux function (3) with asymmetry parameter κ , which has good qualitative agreement with experimental results.

Global Measurements

The segregation time scales linearly with the global volume fraction of small particles, as shown in figure 4. When there are more small particles in the mixture, the segregation is slower. The asymmetric flux function reproduces the linear behaviour, compared to the concave behaviour of the quadratic flux (2). A root-mean squared fit gave $S_r=0.016$ for the quadratic model, and $S_r=0.030$, $\kappa=0.89$ for the cubic model. The asymmetric behaviour is also found in the temporal development of the concentration field shown in figure 5, and is qualitatively found in the numerical integration of the segregation equation (1) using the cubic flux (3). These features are not found with the quadratic flux (2).

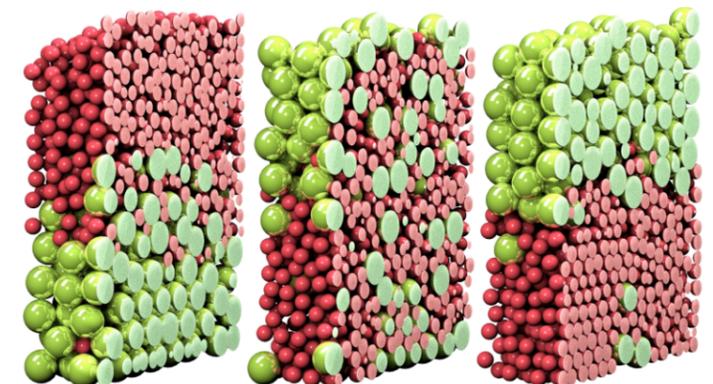
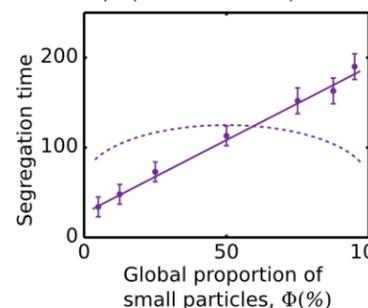


Figure 3 (above): Visualisation of particles positions over time. A few large particles can be seen to be segregating slowly near the bottom.

Figure 4 (below): Total time taken to segregate for mixtures containing different proportions of small particles.



Equations for modelling segregation, where ϕ is the small particle concentration, \hat{t} is the number of cycles, \hat{z} is the scaled height, S_r & D_r are nondimensional segregation & diffusion coefficients respectively. A_κ is a normalisation constant, and κ is the asymmetry parameter:

Segregation equation (1)

$$\frac{\partial \phi}{\partial \hat{t}} - \frac{\partial}{\partial \hat{z}} (S_r F(\phi)) = \frac{\partial}{\partial \hat{z}} \left(D_r \frac{\partial \phi}{\partial \hat{z}} \right)$$
 Quadratic flux function (2)

$$F(\phi) = \phi(1 - \phi)$$
 Cubic flux function (3)

$$F(\phi) = A_\kappa \phi(1 - \phi)(1 - \kappa \phi)$$

Particle Scale Dynamics

Single particle trajectories and root mean squared displacement (RMSD) analysis show that a small particle shows a step-like motion as it falls quickly through gaps in the large particle matrix, whilst, a single large particles segregates three times more slowly, and rises smoothly at a constant speed. The motion of the large particle is governed by the rearrangements of the surrounding particles. These individual dynamics combine to give a characteristic dependence on the local small particle concentration ϕ . The mean vertical displacements per cycle in figure 2 show that large particles are less likely to segregate at high ϕ compared to small particles segregating at low ϕ . The greatest displacement for large particles also occurs at intermediate ϕ , showing that large particles segregate quickest when in proximity to other large particles.

Parallels With Other Physical Environments

These insights give a new understanding of segregation in sheared systems, with the dynamic behaviour of the two species being inherently different. The experimentally determined cubic flux (3) models the asymmetric behaviour and gives good agreement on both bulk and particle scales. This draws parallels with the use of asymmetric flux functions to model asymmetry in sedimentation, traffic flow and diffusion across membranes. The commonality between these processes is their discrete nature, but interestingly, size segregation is the only process that consists of two discrete species.

Figure 2: Mean vertical displacements per cycle as a function of the local small particle concentration. The behaviour is reproduced using the cubic flux ($S_r=0.015$), shown with solid lines. The quadratic flux ($S_r=0.015$) is shown with dashed lines. The values of S_r were adjusted for $\theta_{max}=10^\circ$.

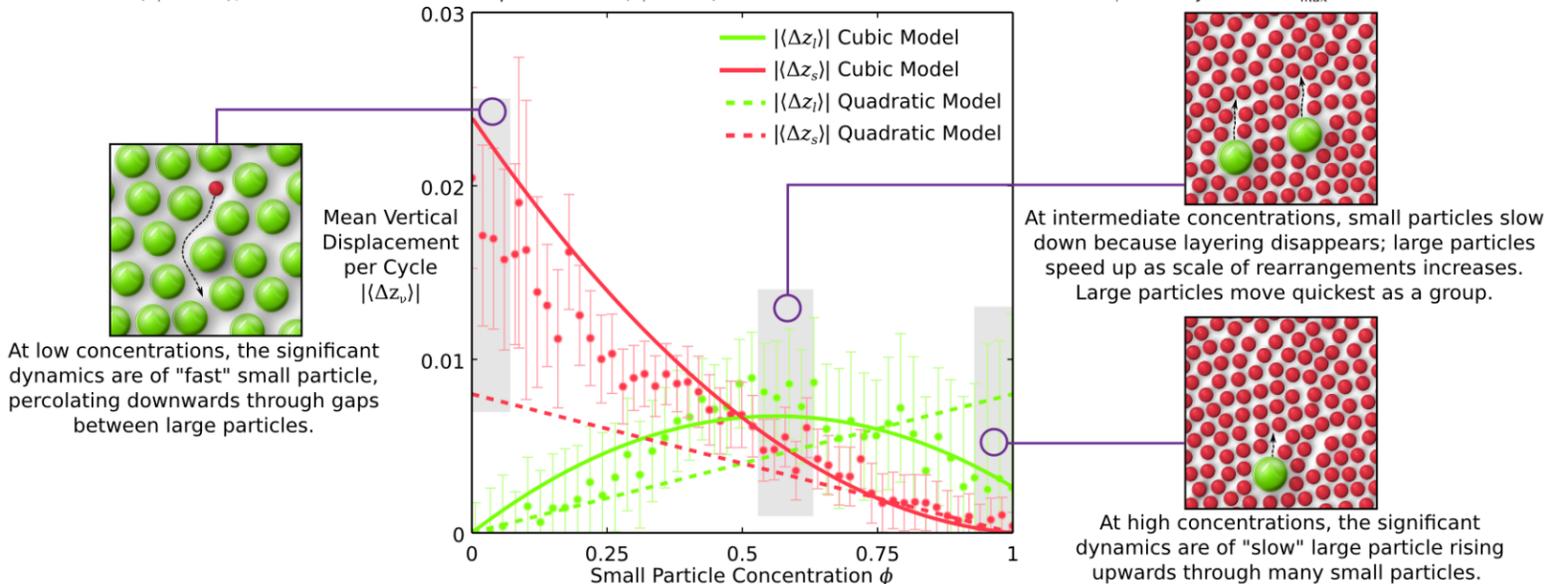


Figure 5 (below): Temporal development of the concentration field, comparing experimental results with numerical integration of equation (1) using the cubic flux function. Parameter values were $S_r=0.030$ and $S_r/D_r=29.6$.

