MATH10242 Sequences and Series: Exercises for Week 8 Tutorials

For your tutorial in the week beginning 16th March.

Question 1: Using L'Hôpital's Rule, or otherwise, find the limit of the sequences

(i) $\left(\dfrac{\ln(7n^{\frac{1}{4}}-2)}{\ln(n+1)}\right)_{n\in\mathbb{N}}$ (ii) $\left(\dfrac{e^{e^n}}{e^n}\right)_{n\in\mathbb{N}}$

(iii) $\left(\dfrac{1-e^{-n}}{2-e^{-2n}}\right)_{n\in\mathbb{N}}$ (iv) $\left(\dfrac{1-e^{n}}{2-e^{2n}}\right)_{n\in\mathbb{N}}$

Question 2: (i) Use L'Hôpital's Rule to show that $\dfrac{(\ln n)^2}{n} \to 0$ as $n \to \infty$.

(ii) Show by induction that for any $k \in \mathbb{N}$, $\dfrac{(\ln n)^k}{n} \to 0$ as $n \to \infty$.

Question 3: (i) Using the formula $(x-y) = \dfrac{(x-y)(x^2+xy+y^2)}{(x^2+xy+y^2)} = \dfrac{(x^3-y^3)}{(x^2+xy+y^2)}$ or otherwise, find

$$\lim_{n\to\infty} \sqrt[3]{n^3+n^2} - n.$$

(ii) Show that $[\sqrt[3]{n^3+n^2}\,] = n$.

(iii) Using subsequences show that $[\sqrt[3]{n}\,] - \sqrt[3]{n}$ does not have a limit.