MATH10242, Sequences and Series: markers' comments on the June 2018 exam

Overall, the exam was done well, with generally good answers to "method" questions where there had been opportunity to practice, but weaker answers (in general) to the "theory" questions which demand sound understanding and the ability to present reasoning clearly and coherently. A lot of answers were unclear or poorly written, with the logic connecting statements wrong or simply missing. Students must learn that it's not enough to have a rough idea how it works in your head if you can't translate that into clear, unambiguous maths on the page. Particularly of concern was the number of students who couldn't correctly use basic symbols like $=$, $\iff$, $\Rightarrow$, and $\to$: each of these was misused by some people to mean one of the others.

A1 was answered well in general, although a lot of students wrote out the definition for a sequence to be *eventually* increasing / *eventually* strictly decreasing instead. Almost all students could state a version of the monotone convergence theorem. A fair amount of students seemed to be confused by the notation of sequences.

A2 was mixed. Although most students could do the algebraic manipulation of inequalities to obtain a suitable N, almost all students failed to point out the reverse implication to show that this N worked, and failed to get the full 6 marks.

A3 was answered fairly well. Most students could calculate the limits through inspection. The arguments given tended to be too informal sometimes. [Ans: (a) 0; (b) $\frac{1}{3}$]

A4 A good number of students did not use the algebra of limits to simplify after their first application of L'Hopital, resulting in rather complicated expressions (and errors) after further differentiations. [Either differentiate 3 times, splitting the limit each time to simplify (and remembering to note that the conditions for L'Hopital are satisfied) or use the Algebra of Limits (which can be justified retrospectively) to take the cube outside the limit and then L'Hopital is needed only once.]

A5 Many students failed to apply appropriate tests and/or drew unjustified/wrong conclusions from attempted applications of tests. A lack of enough practice seemed to be showing here. Note that the Alternating Series Test requires the sequence to be (eventually) decreasing, not just to converge to 0. [Ans: (a) Divergent, by the Ratio Test or Comparison; (b) Divergent by Comparison, (or the Integral Test); (c) Convergent - Alternating Series]

A6 was done well. But do remember to use the definition of infinite sum as the limit of finite partial sums.

B7 (i) Many students remembered the core of the argument but did not set up well: ϵ and η appearing from nowhere; no mention on the fact that the estimate would be valid only for $n \geq N$ for appropriately chosen N; associating N with the bounded sequence rather than with the null sequence.

(ii) Almost everyone did this correctly, giving a counterexample.

(iii), (iv) were done very well.

B8 Generally done quite well, but many students had problems setting up and carrying through an induction argument (especially one which contained two statements). Do make sure you know how to write out induction arguments. [The limit is 1.]

Also, make sure to carefully distinguish between what the question is telling you and what it is asking you to prove (quite a few students took something which they were asked to prove as being given).

B9 Quite a number of students had a confused or wrong definition of the radius convergence. Many students gave little or no justification for what they wrote - not a lot, but some, is required for full marks. [Ans: (i) RoC$= 5$; (ii) RoC$= \infty$; (iii) IoC$= (-1, 1]$.]