# MATH10242, Sequences and Series: markers' comments on the May 2017 exam

June 15, 2017

Overall, the exam was done well, with generally good answers to "method" questions where there had been opportunity to practice, but weaker answers (in general) to the "theory" questions which demand sound understanding and the ability to present reasoning clearly and coherently. Since there were rather few compulsory "theory" parts, the marks did come out rather high.

Questions A1-A4: No question was done consistently well, none was done consistently badly. Some people gave an incoherent answer to A1 but then near-perfect solutions to the rest, and some gave textbook definitions in A1 and then made many errors on the rest.

A lot of answers were unclear or poorly written, particularly with the definitions and the more logic-sensitive parts. Students need to learn it's not enough to have a rough idea how it works in your head if you can't translate that into clear, unambiguous maths on the page. Particularly of concern was the number of students who couldn't correctly use basic symbols like $=$, $\iff$, $\Rightarrow$, and $\rightarrow$: each of these was misused by some people to mean one of the others.

Specific comments:

A1: Quite a few problems with quantifiers; especially people using vague English; e.g., "for $\epsilon$" but does this mean "for some $\epsilon$ or "for all $\epsilon$"? (indeed, some students seemed to use it for both meanings within the same question!). In part (b), a lot of people gave the definition for "bounded" instead of bounded above. Also, a small but significant number got "bounded above" confused with "increasing" or else gave a (frequently incorrect) statement of the Monotone Convergence Theorem in lieu of the definition.

A2: Most students were able to get 3 or 4 marks on this, but not many got the full 6. The algebraic manipulations were typically fine; the problem was almost invariably the logic - having forward implications when backwards ones or iffs were needed. A number also forgot to use the integer part function in order to make $N$ a natural number.

A3: Most students managed to make a good attempt at this question; almost all knew how to approach it, though quite a few slipped up to a greater or lesser extent on the details. Many people neglected to mention that they were using the Algebra of Limits. A lot of people also made careless mistakes with the notation e.g., writing $\lim_{n\to\infty}(a_n) = a'_n$, where $a'_n$ is some algebraic simplification of $a_n$; or manipulating $a_n$ without lim in front, but then writing $=$ instead of $\rightarrow$ when taking the limit.

A4: Probably the best done of these four questions. It was reassuring how many students remembered that there are conditions to check before applying l'Hopitals Rule, and most were able to differentiate correctly and work out how to deal with the ensuing limit (though often after a bit of fruitless rearranging). The main problems were failing to mention the Algebra of Limits and notational errors. Some differentiated $e^x + x$ as $e^x$, and $1/x$ as $\log(x)$; those who made the latter mistake then very frequently claimed that $\log(n) \to 0$ in order to get the expected answer at the end, rather than realising they must have made a mistake. A small but significant number misapplied the AoL Theorem either saying $\lim(a_n b_n) = \lim(a_n)\lim(b_n)$ without first establishing that the second two limits exist, or else applying it to part of a complicated expression, then rearranging the remaining part, before applying again. Students should note that after taking a limit as $n \to \infty$, you shouldn't have any $n$s left in the expression you get!

A5: (a) This was done pretty well by most students. The most common mistakes were not including the comment that the function is decreasing, continuous and positive, and also failing to write down the limit as $N \to \infty$, for the integral with the limit $N$.

(b) Also, no large problems in this one, though some tried other methods without getting anywhere.

(c) Comparison worked fine. Most common mistakes were writting that $1/n^2$ converges instead of the fact that the sum converges and, most important, that the sum(original series)<sum($1/n^2$).

A6: Almost everyone found the fractions correctly, and worked out the sums up to a certain point. The most common mistake was not writing down the *finite* partial sum $\sum_1^N = \ .....$ , rearranging and then letting $N \to \infty$. Many kept writing $\sum_1^\infty$.

B7: (i) Many people invoked the algebra of limits at some point, rather than working, as the question asked, from the definition of convergence, so lost some or all marks.

(ii) was well done.

(iii) (a) Again, many people used the algebra of limits rather than working from the definition of convergence. For part (b) there were some nice answers; but a few people essentially replaced the "$-$" in the definition of $a_n^*$ by "$+$", so didn't answer the question asked.

B8, Most students used the Monotone Convergence Theorem correctly.

It was a common mistake for students to write the proof without connecting the steps (by using words such as if, iff, then, so, etc.).

Many students incorrectly resolved the inequality $ab > 0$ into conditions on $a$ and $b$.

Many students used the Algebra of Limits without mentioning it.

B9 Most students did this question and, in general, it was answered well.

- (i) was answered very well. The most common mistake was leaving out "and diverges for $|x| > R$" but even this was rare.
- (ii) again answered well. The most common slip-ups were missing modulus signs and not making it clear that a limit is being taken (e.g. using $=$ instead of $\to$).
- (iii) was found hardest with a large minority struggling to get beyond evaluating the limit of $|\frac{n\ln(n)}{(n+1)\ln(n+1)}|$. Common errors for those who answered were:
  - Same as part (ii)
  - Probably the hardest mark to get: not fully justifying why $|\frac{n\ln(n)}{(n+1)\ln(n+1)}| \to 1$ (use of l'Hopital's rule was required for full marks)
  - When applying the integral test to check convergence at $x = -1$, many students did not show understanding that the integral is in fact a limit (e.g. often writing $\int = \ln(\ln(\infty)) - \ln(\ln(2))$ ).