~~9/3/20~~ (1)

9/8/20 (1)

- Exactly one of these statements is true.

- If $(a_n)_{n\in\mathbb{N}}$, $(b_n)_{n\in\mathbb{N}}$, $(c_n)_{n\in\mathbb{N}}$ are convergent sequences with limits a, b, c respectively, and if, for all n, we have $a_n < b_n < c_n$, then $a < b < c$. ~~Ad~~ False eg $a_n = -\frac{1}{n}$ $b_n = 0$ $c_n = \frac{1}{n}$

- The sequence $7, 7^{1/2}, 7^{1/3}, 7^{1/4}, \ldots, 7^{1/n}, \ldots$ converges to 1. True

- If $(a_n)_{n\in\mathbb{N}}$, $(b_n)_{n\in\mathbb{N}}$, $(c_n)_{n\in\mathbb{N}}$ are convergent sequences with limits a, b, c respectively, then $\left(\frac{a_n}{b_n + c_n}\right)_{n\in\mathbb{N}}$ is convergent and has limit $\frac{a}{b+c}$. ~~Ab~~ False eg if $b + c = 0$

Chpt 6 Subsequences

Given a sequence $(a_n)_n$, a subsequence is what we get by deleting (possibly infinitely many) terms, as long as infinitely many terms remain.

Defn Given a sequence $(a_n)_{n\in\mathbb{N}}$, a subsequence $(b_n)_n$ is given by a sequence $1 \leq k_1 < k_2 < k_3 < \cdots < k_n < \cdots$ of integers by $b_n = a_{k_n}$.

eg Take $k_n = 3n$ ($k_1 = 3$ $k_2 = 6$ $k_3 = 9$, ...) and the corresponding subsequence $(b_n)_n$ is

$$\begin{array}{ccccccccc} a_3 & a_6 & a_9 & a_{12} & \cdots & a_{3n} & \cdots \\ \| & \| & \| & \| & & \| & \\ b_1 & b_2 & b_3 & b_4 & \cdots & b_n & \cdots \end{array}$$

eg If $a_n = (-1)^n$ then the subsequence $(\overset{b_n}{\overset{\|}{a_{2n}}})_n$ is the constant sequence 1 $\doteq c_n$

and the subsequence $(a_{2n-1})_n$ is the constant sequence -1.

eg 6.1.2 The sequence $(n^2)_{n\in\mathbb{N}}$ ($a_n = n^2$) has, as a subsequence the sequence $(4^n)_n$.

(Because $4^n = 2^{2n} = (2^n)^2 = a_{2^n}$ – so this is the subsequence given by $k_n = 2^n$)

6.1.3 Given a sequence $(a_n)_{n\in\mathbb{N}}$ a subsequence $(b_n = a_{k_n})_{n\in\mathbb{N}}$ is: 9/3/20 (3)

1) convergent to l if $a_n \to l$ as $n\to\infty$
ii) tending to ∞ if $a_n \to \infty$ as $n \to \infty$
iii) ~~ten~~ going to $-\infty$ if $a_n \to -\infty$ as $n\to\infty$.

Proof 1) If (a_n) converges, with limit l, then, given $\varepsilon > 0$, there is N s.t.

$\forall n \geq N \quad |a_n - l| < \varepsilon$ (because $a_n \to l$ as $n\to\infty$)

Since, note, $k_n \geq n$, we have

$\forall n \geq N \quad |a_{k_n} - l| < \varepsilon$ (because $n \geq N$ implies $k_n \geq N$)

6.1.3 says that, if a sequence converges, then every subsequence converges to the same limit.

So if $(a_n)_n$ is a sequence which has two subsequences which converge to different limits, then $(a_n)_n$ does not converge.

Ex 6.1.4 $a_n = (-1)^n + \frac{1}{n^2}$

The subsequence $a_{2n} = 1 + \frac{1}{n^2} \to 1$ as $n\to\infty$

but the subsequence $a_{2n+1} = -1 + \frac{1}{n^2} \to -1$ as $n\to\infty$

Hence $(a_n)_n$ does not converge.

Ex 6.1.5 $\quad a_n = \frac{n}{4} - \left[\frac{n}{4}\right]$ 9/3/20 (4)

Consider $a_{4n} = \frac{4n}{4} - \left[\frac{4n}{4}\right] = n - [n] = n - n = 0$

$\longrightarrow 0$ as $n \to \infty$

but $a_{4n+1} = \frac{4n+1}{4} - \left[\frac{4n+1}{4}\right] = n + \frac{1}{4} - \left[n + \frac{1}{4}\right]$

$= n + \frac{1}{4} - n = \frac{1}{4} \longrightarrow \frac{1}{4}$ as $n \to \infty$

Hence $(a_n)_n$ is not convergent

Ex 6.1.7 $\quad a_n = [\sqrt{n}] - \sqrt{n}$

Consider the subsequence

$a_{n^2} = [\sqrt{n^2}] - \sqrt{n^2} = [n] - n = n - n = 0$