Treatment of problems with memory – parameters, perturbations, and singular perturbation

Manchester Applied Mathematics and Numerical Analysis Seminars
January 20, 1999, 4 pm
Lecture Theatre D13 Material Science

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Abstract

Many real-life problems that are often modelled by differential equations in reality involve a significant time lag, or memory effect, that can be reflected in a more refined model. Examples of such models are delay differential equations, such as

$$y'(t) = F\Big(t, y(t), y(\alpha(t))\Big), \quad t \geq t_0, \qquad (i)$$

$$y(t) = \psi(t), \quad t \in [t_{min}, t_0], \qquad (ii)$$

where $\min_{t \geq t_0} \alpha(t) = t_{min}$.

The general class of problem can be represented as $y'(t) = \mathfrak{F}(t, y_{[t_{min},t]}(\cdot))$, for $t \geq t_0$; wherein $y_{[t_{min},t]}(\cdot)$ is the restriction of $y(\cdot)$ to the domain $[t_{min}, t]$, for some $t_{min} \leq t_0$ and where $y_{[t_{min},t_0]}(\cdot) = \psi_{[t_{min},t_0]}(\cdot)$. This general form is too abstract for us to distinguish features (fading or persistent memory, or constant or state-dependent lag, for example) that are important in mathematical modelling, and in the analysis and in the numerical treatment of real-life problems. However, in contrast, numerical analysts have often resorted (when addressing the numerics of such evolutionary problems) to a discussion of a simple test equation such as

$$y'(t) = \lambda y(t) + \mu y(t - \tau_\star), t \geq t_0, \quad y(t) = \psi(t) \text{ for } t \in [-\tau_\star, t_0]\ (\tau_\star \geq 0). \qquad (iii)$$

The parameters in (iii), and in similar paradigms for (i), make the equations into flexible friends, allowing us to illustrate many phenomena, even though such equations have obvious limitations.

We shall discuss how such parameterized equations relate to wider classes of problems, how they may be used to gain insight into numerical algorithms, and, if time permits, indicate how (in some cases) they have a rôle in producing quantitatively accurate models for real-life phenomena.

Keywords Memory, lag, delay differential equations, causal problems, discontinuities, stability, stiffness, singular perturbation, bifurcation, algorithms, parameter estimation