

# MT1612: EXAMPLE SHEET<sup>1</sup> III (for Feb. 24, 1999)

1.) Use Euler's theorem to put the following expressions into polar form (using the complex exponential)

(i)  $z = -5i$ ,

(ii)  $z = -1 + i$ ,

(iii)  $z = -2 - 2\sqrt{3}i$ ,

(iv)  $z = (-\sqrt{2} - \sqrt{2}i)^7$ .

2.) Using the relations between the complex exponential,  $\sin / \cos$  and  $\sinh / \cosh$  determine the real and imaginary parts of

(i)  $e^{2+5\pi i/6}$ ,

(ii)  $\cos(i)$

(iii)  $\cosh(\pi i)$

(iv)  $\sin(\pi/3 - i)$  [Hint:  $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$ ].

3.) Use the relations between the complex exponential and  $\sin / \cos$  to

(i) write  $\cos^3 \varphi$  in the form  $a_1 \cos \varphi + a_3 \cos(3\varphi)$ ,

(ii) write  $\sin^4 \varphi$  in the form  $a_0 + a_2 \cos(2\varphi) + a_4 \cos(4\varphi)$ .

4.) Find *all* values of

(i)  $\ln(-3)$ ,

(ii)  $\ln(-1 - i\sqrt{3})$ .

(iii) What is the principal value of  $\ln i$ ?

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