MT35001: SOLUTION FOR EXAMPLE SHEET[1] I

1.) Which one of these equations in index notation are valid? Remember the summation convention!

a) $c = a_i b_i$ (OK, this is the dot product $c = \mathbf{a} \cdot \mathbf{b}$)

b) $c = a_{ij} b_i$ (Wrong, the free index j doesn't appear on LHS)

c) $c_i = a_{ij} b_i$ (Wrong, the indices on LHS and RHS don't match)

d) $c_i = a_{ij} b_j$ (OK, this is the matrix vector product with the matrix $\underline{\mathbf{a}}$: $\mathbf{c} = \underline{\mathbf{a}}\mathbf{b}$)

e) $c_i = a_{ji} b_j$ (OK, this is the matrix vector product with the transposed matrix $\underline{\mathbf{a}}$: $\mathbf{c} = \underline{\mathbf{a}}^T\mathbf{b}$)

f) $\sigma_{ij} = \alpha_{ij} T + E_{ijkl} e_{kl}$ (Correct – meet your first 4th order tensor. By the way: this is the constitutive equation for a linearly elastic solid incl. temperature variations)

g) $\sigma_{ij} = \alpha_{kl} T_i + E_{ijkl} e_{ij}$ (Wrong, the indices of all terms are different)

h) $k_{ijkl} = a_i b_{kl} c_{njm} d_{mn} + e_{ik} e_{jn} f_{nl}$ (Messy, but correct)

2.) Using a comma to denote partial differentiation (e.g. $\partial u / \partial x_2 = u_{,2}$), transform the following expressions into index notation:

a) $\nabla u(x_1, x_2, x_3) \to u_{,i}$

b) $\underline{\mathbf{A}} = \nabla \mathbf{u}(x_1, x_2, x_3) \to a_{ij} = u_{i,j}$

c) $\nabla \cdot \mathbf{u}(x_1, x_2, x_3) = f(x_1, x_2, x_3) \to u_{i,i} = f$

d) $\nabla^2 u(x_1, x_2, x_3) = f(x_1, x_2, x_3) \to u_{,ii} = f$

e) $\nabla^2 \mathbf{u}(x_1, x_2, x_3) = \mathbf{f}(x_1, x_2, x_3) \to u_{i,jj} = f_i$

3.) a) Show that $\frac{\partial x_i}{\partial x_j} = \delta_{ij}$: Cartesian coordinates are independent of each other, so $\frac{\partial x_i}{\partial x_j} = 1$ if $i = j$ and 0 if $i \neq j$.

b) Show that $\delta_{ii} = 3$: Using the summation convention this expands as $\delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33}$, which, given the properties of the Kronecker delta, is equal to three.

c) For arbitary vectors u_i and v_i show that

$$S_{ij} = u_i v_j + u_j v_i$$

is symmetric (i.e. $S_{ij} = S_{ji}$) and that

$$T_{ij} = u_i v_j - u_j v_i$$

is antisymmetric (i.e. $T_{ij} = -T_{ji}$). Exchange the indices i and j and re-arrange the terms.

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