$$\boxed{\begin{gathered}\rho\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=\rho F_i-\frac{\partial p}{\partial x_i}+\mu\frac{\partial^2 u_i}{\partial x_j^2}\\ \frac{\partial u_j}{\partial x_j}=0\end{gathered}}$$

Nonlinear $\rightarrow$ hard to solve! But the nonlinear terms disappear in certain circumstances.

§5 Parallel flows

Assume: flow is uni-directional

Choose coordinates s.t.

$$u_1 = u \neq 0$$

$$u_2 = u_3 = v = w = 0$$

$$\underline{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$(x_1, x_2, x_3) = (x, y, z)$$

Note: u can still vary as a fct of $\underline{x}$ & t.

Consequences:

continuity:

$$\frac{\partial u_j}{\partial x_j}=0 \; : \; \frac{\partial u}{\partial x}+\cancel{\frac{\partial v}{\partial y}}+\cancel{\frac{\partial w}{\partial z}}=0$$

$$\Rightarrow u = u(y,z,t)$$

Use this in x-comp. of the mom. eqn:

$$\rho\left(\frac{\partial u}{\partial t}+\cancel{u\frac{\partial u}{\partial x}}+\cancel{v\frac{\partial u}{\partial y}}+\cancel{w\frac{\partial u}{\partial z}}\right)=$$

$$=\rho F_x-\frac{\partial p}{\partial x}+\mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\boxed{\rho\frac{\partial u}{\partial t}=\rho f_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)} \quad (1)$$

y-comp:

$$\rho\left(\cancel{\frac{\partial v}{\partial t}}+u\cancel{\frac{\partial v}{\partial x}}+v\cancel{\frac{\partial v}{\partial y}}+w\cancel{\frac{\partial v}{\partial z}}\right)=$$

$$=\rho f_y-\frac{\partial p}{\partial y}+\mu\,\cancel{\nabla^2 v}$$

$$\boxed{\rho f_y=\frac{\partial p}{\partial y}} \quad (2)$$

similarly:

$$\boxed{\rho f_z=\frac{\partial p}{\partial z}} \quad (3)$$

Note (1)-(3) are linear.

Special case: No body force

In that case:

(2) & (3) $\Longrightarrow$ $p(x,t)$ (at most!)

into (1): $u = u(y, z, t)$ $\quad$ (u

$p = p(x, t)$

$$\underbrace{\rho \frac{\partial u}{\partial t}}_{\text{fct of } (y,z,t)} = \underbrace{-\frac{\partial p}{\partial x}}_{\text{fct of } (x,t)} + \underbrace{\mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)}_{\text{fct of } (y,z,t)}$$

This can only be accomodated if $\frac{\partial p}{\partial x}$ is not a fct. of x.

$$\frac{\partial p}{\partial x} = G(t)$$

Parallel flow eqns without body force

$$\frac{\partial u}{\partial t} = -\frac{G(t)}{\rho} + \nu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

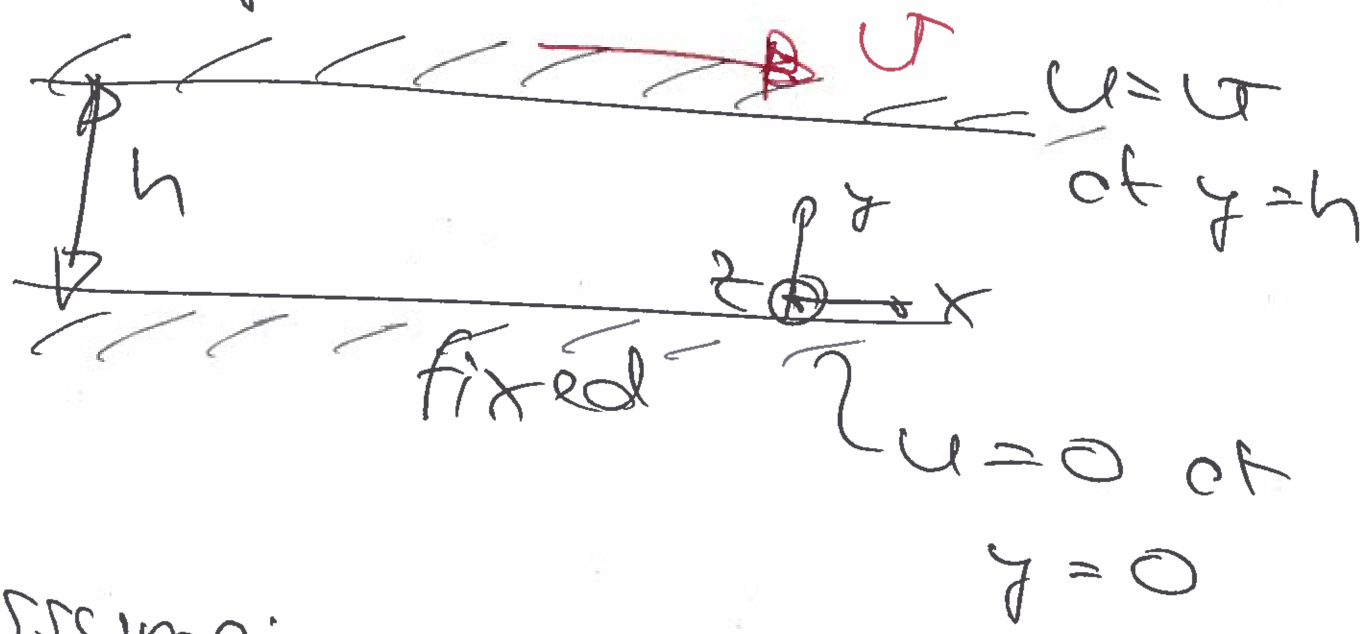
$\nu = \frac{\mu}{\rho}$

Kinematic viscosity.

$$\frac{\partial p}{\partial x} = G(t) \;; \quad v = w = 0$$

Example: Couette flow

Flow between parallel infinite plates. Upper plate moves with veloc. U to the right:



Assume:

- parallel flow in x-direction
- $u(y,\cancel{z},\cancel{t}) = u(y)$
- $G(t) = 0$

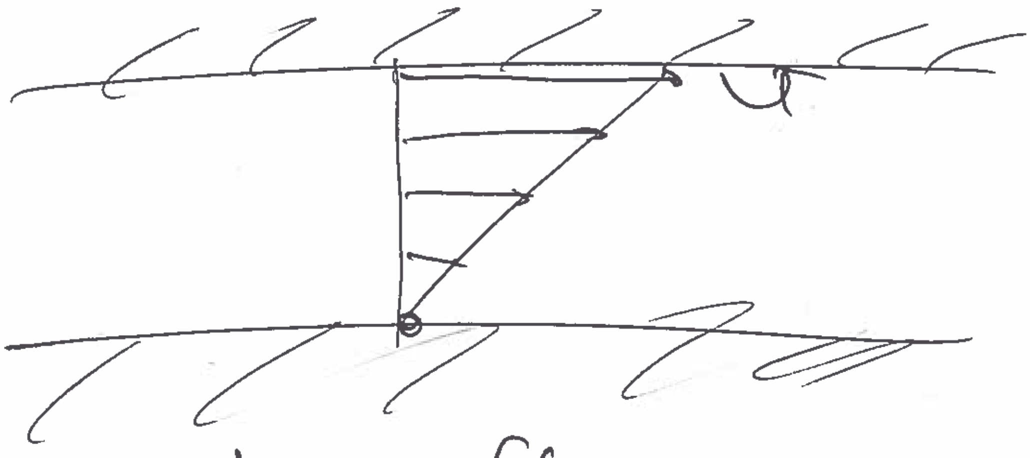
$$\cancel{\frac{\partial u}{\partial t}} = -\cancel{\frac{G}{\rho}} + \nu\left(\frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

$$\frac{d^2 u}{dy^2} = 0 \Rightarrow u(y) = Ay + B$$

<u>2 BC:</u>

$$u(y=0) = 0$$
$$u(y=h) = U$$

$$\Rightarrow u(y) = U\frac{y}{h}$$



sheer flow.