Cauchy's eqn

$$\frac{\partial \tau_{ij}}{\partial x_j} + \rho F_i = \rho\left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}\right)$$

Evolution eqn. for u_i but what is τ_{ij} ?

$\Longrightarrow$ Constitutive eqn.
(we only consider incompr. fluids)

Observations:

Fluids:

- can generate hydrostatic pressures
- have a resistance to shear (viscosity)
- do not generate internal stresses when subjected to rigid body motions.

A wide range of fluids (Newtonian fluids) behave according to:

$$\tau_{ij} = \underbrace{-p\,\delta_{ij}}_{\text{hydrostatics}} + \underbrace{2\mu\,\varepsilon_{ij}}_{\text{stress due to deformation}}$$

μ = "dynamic viscosity" has to be measured

$$\tau_{ij} = -p\,\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

into Cauchy:

$$\rho\frac{Du_i}{Dt} = \rho f_i + \frac{\partial}{\partial x_j}\left(-p\,\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right)$$

$$= \rho f_i - \frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j^2} + \cancel{\mu\frac{\partial}{\partial x_i}\left(\frac{\partial u_j}{\partial x_j}\right)}$$

because $\operatorname{div} \underline{u} = 0$

$$\boxed{\rho\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=\rho f_i-\frac{\partial p}{\partial x_i}+\mu\frac{\partial^2 u_i}{\partial x_j^2}}$$

need to add the continuity eqn

$$\boxed{\frac{\partial u_j}{\partial x_j}=0}$$

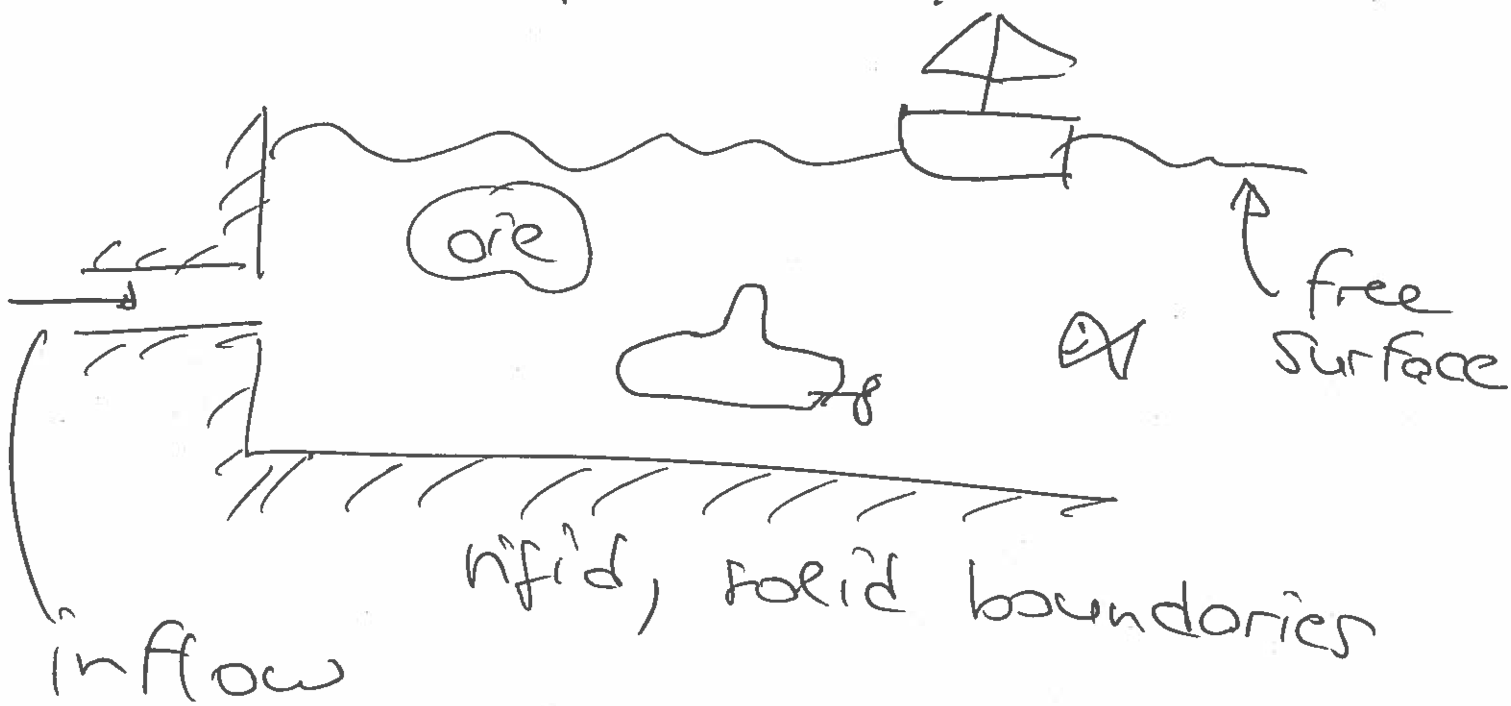
$\Rightarrow$ Navier Stokes !! (momentum eqns)

<u>N. St. eqns:</u>

A system of four nonlinear coupled PDEs; second order in space; 1st order in time for the three veloc. comp. u_i & the pressure p.

§4 Boundary & initial conditions

General problem:



Initial conditions

Need to specify

$u_i(x_j, t=0)$

for time-dependent problems

(no IC for pressure!)

Boundary conditions

(i) inflow/outflow BCs

$u_i = v_i$ (given) on inflow/outflow boundary

(ii) on solid surfaces

"No slip & no penetration"

Solid velocity = fluid velocity

given

$u_i = v_i$ — solid velocity

Special case: Rigid, stationary walls $\Rightarrow u_i = 0$

(iii) free surfaces

we need two conditions

- kinematic BC
- traction BC.