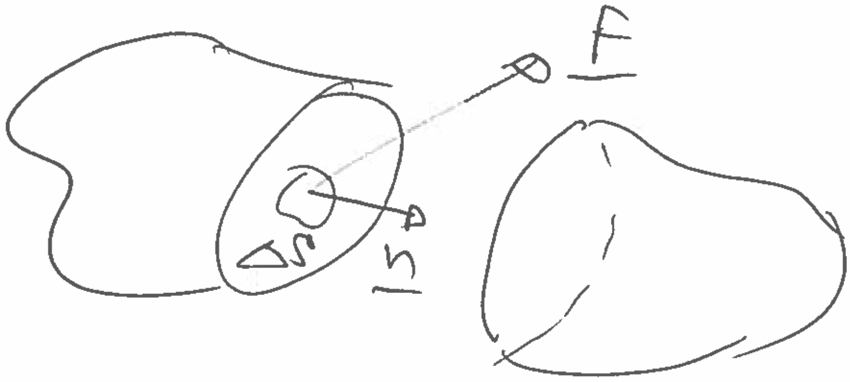


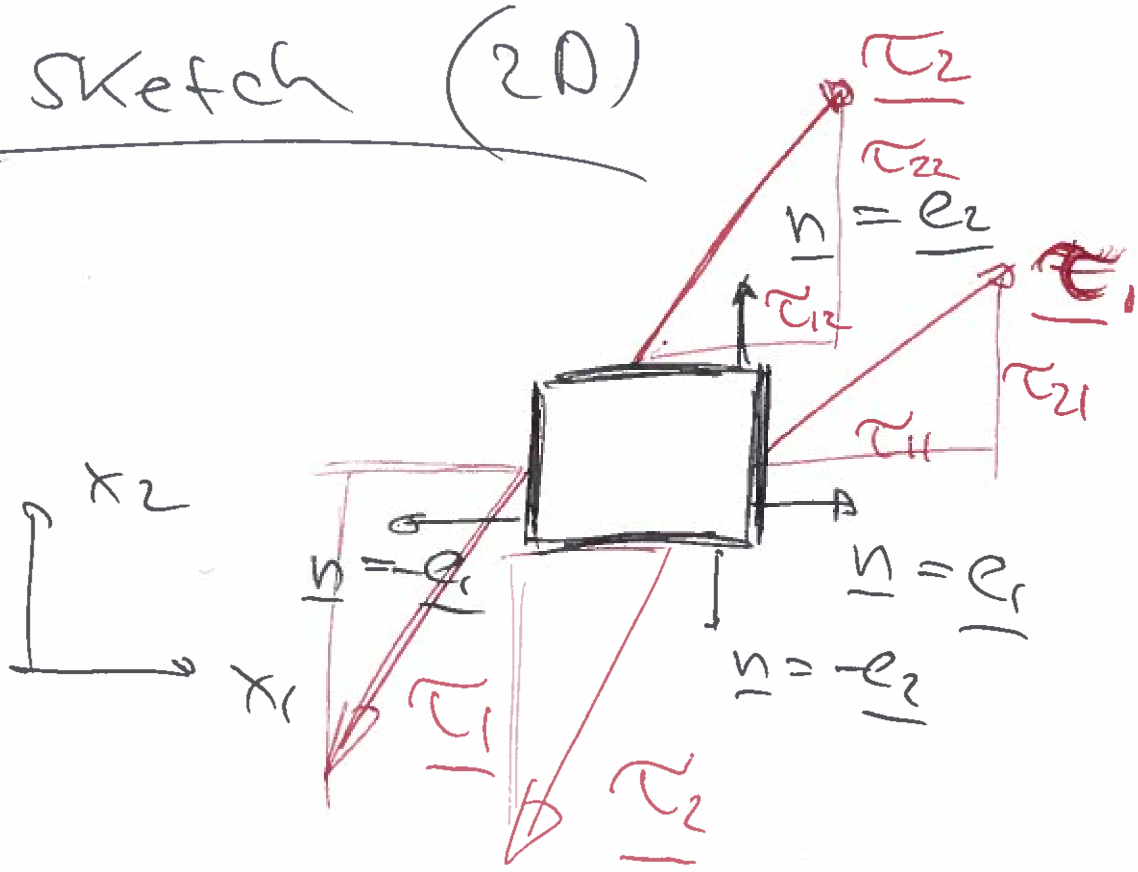
stress/traction

$$\underline{t} = \lim_{\Delta S \to 0} \frac{\underline{F}}{\Delta S} \quad \text{(vector!)}$$

$$t_i = \underbrace{\tau_{ij}}_{\text{stress tensor}} n_j$$

τ_{ij} represents the traction/stress component in the pos. i-th direction on the face $x_j = \text{const}$ whose outer unit normal points in the positive x_j-direction.

Sketch (2D)



Particular stress states:

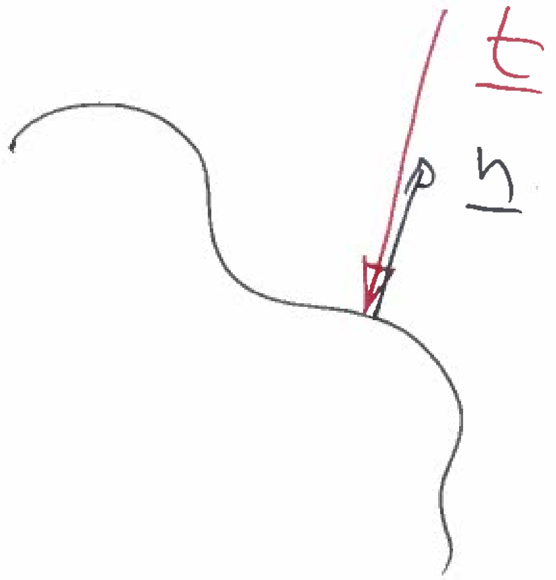
(i) Hydrostatic pressure

$$\tau_{ij} = -p\,\delta_{ij}$$

implies that the stress is always normal to the plane & uniform in all directions

$$t_i = \tau_{ij} n_j = -p\,\underbrace{\delta_{ij} n_j} = -p n_i$$

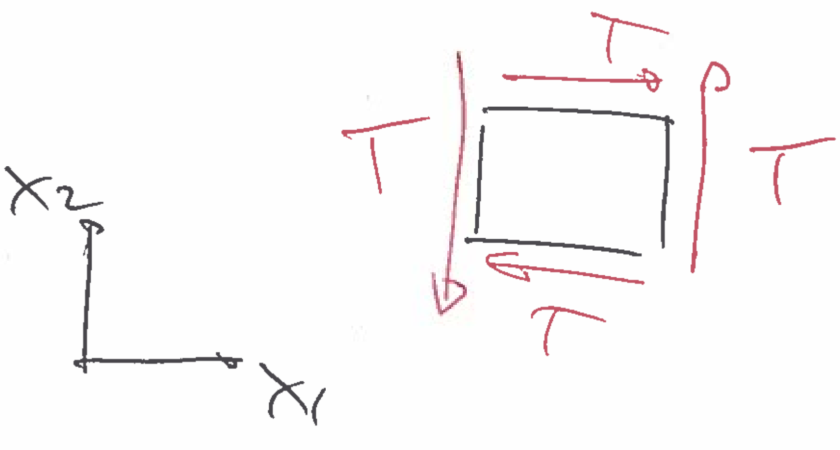
$$\underline{t} = -p\,\underline{n}$$



(ii) pure shear stress

e.g. $\tau_{12} = \tau_{21} = T$

$\tau_{11} = \tau_{22} = 0$



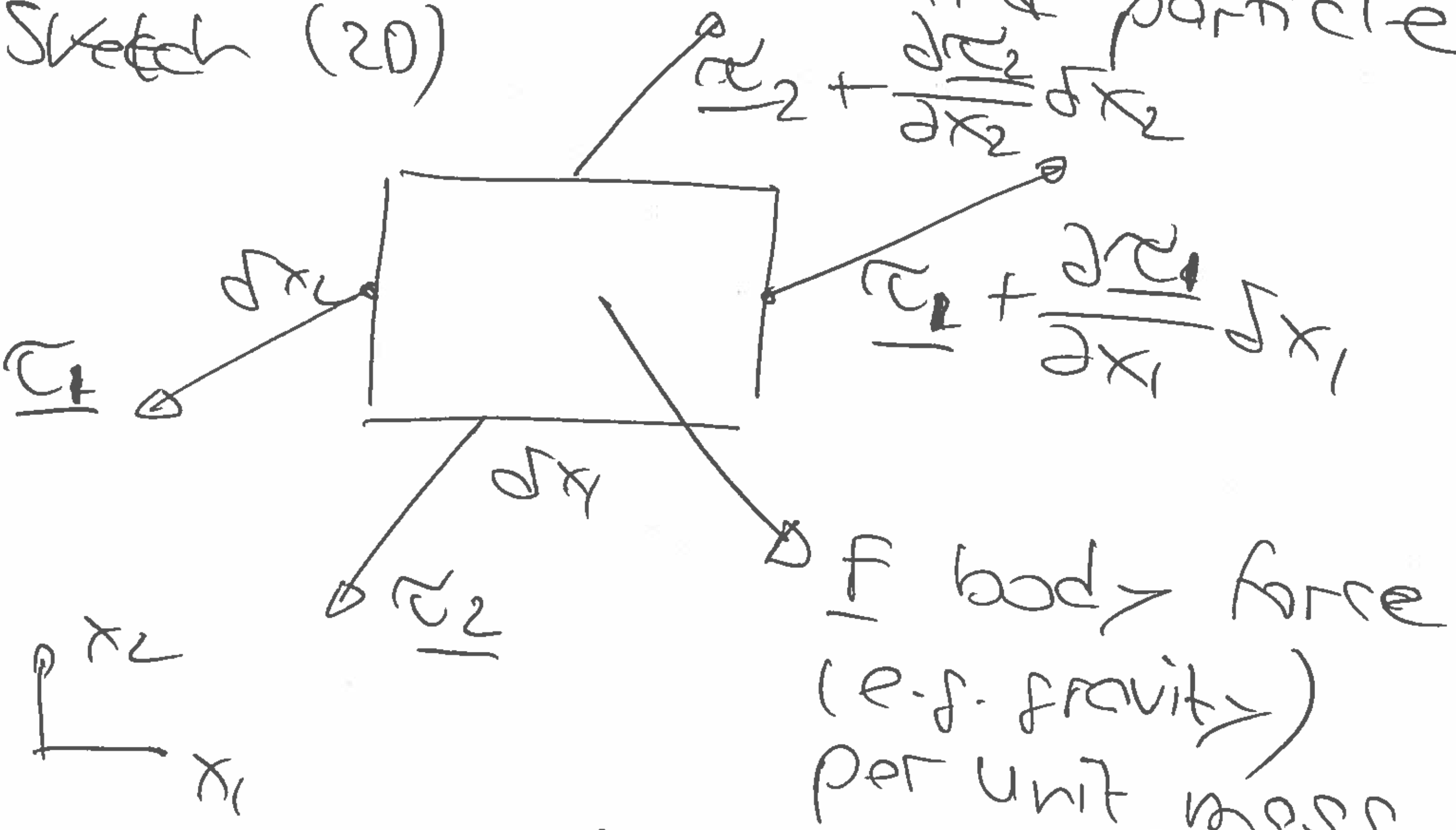
3. Equilibrium of forces

Newton's law:

$$\sum \text{forces} = \text{mass} \times \text{accel.}$$

for a small fluid particle.

Sketch (2D)



(use only increment in tractions/stresses):

$$\frac{\partial \tau_1}{\partial x_1} \cancel{\delta x_1 \delta x_2} +$$

$$\frac{\partial \tau_2}{\partial x_2} \cancel{\delta x_2 \delta x_1} +$$

$$\underline{F} \rho \cancel{\delta x_1 \delta x_2} =$$

$$\rho \cancel{\delta x_1 \delta x_2} \frac{Du}{Dt}$$

$$\frac{\partial \tau_j}{\partial x_j} + \underline{F} \rho = \rho \frac{Du}{Dt}$$

$$\boxed{\frac{\partial \tau_{ij}}{\partial x_j} + \rho F_i = \rho \frac{Du_i}{Dt}}$$

(Cauchy's eqn.)

$$\frac{\partial \tau_{ij}}{\partial x_j} + \rho F_i = \rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right)$$

evolution eqn. for u_i once we know what τ_{ij} is!

Body force (per unit mass):
for gravity: $\underline{f} = \underline{g}$.

4. Symmetry of stress tensor

$$\tau_{ij} = \tau_{ji}$$

5. Constitutive eqns & the Navier-Stokes eqns

(we'll restrict ourselves to incompressible fluids)
Const. eqn. provide a link between the stress & the kinematics of deformation.
(phenomenological)