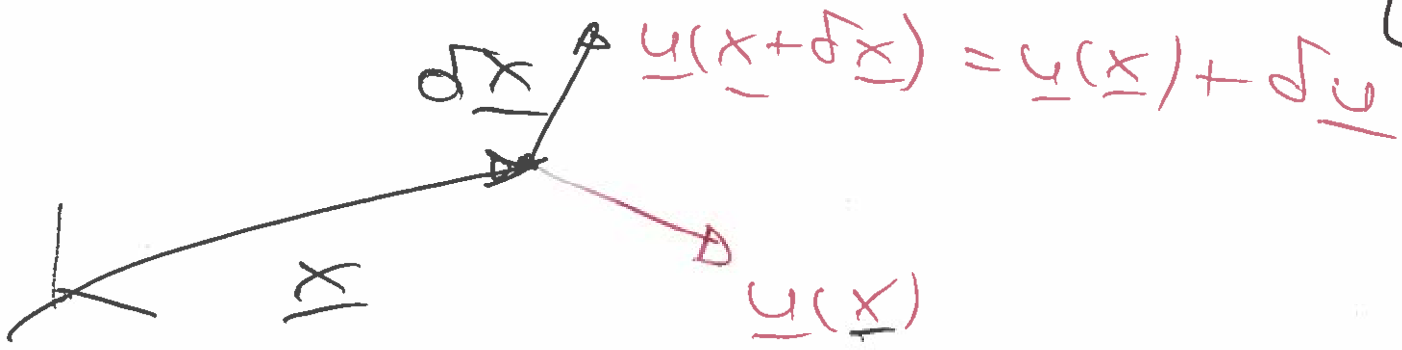
(1

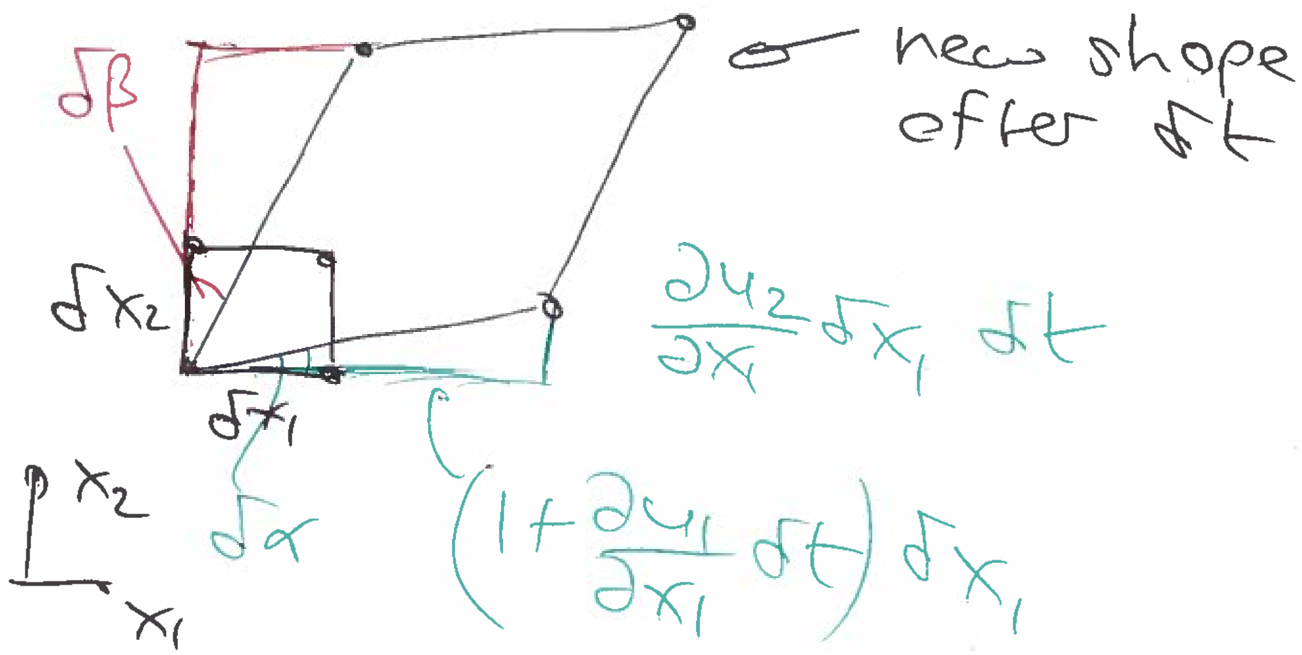


$$\delta u_i = \underbrace{\varepsilon_{ij}\,\delta x_j}_{\text{dilation (stretching) \& shearing}} + \underbrace{\omega_{ij}\,\delta x_j}_{\text{rigid body rotation}}$$

$\varepsilon_{(ii)}$ = rate of strain of material line in x_i-dir.

(ii) Shear rate of strain

(2D sketch)



$$\tan\delta\alpha = \frac{\frac{\partial u_2}{\partial x_1}\cancel{\delta x_1}\,\delta t}{\left(1+\frac{\partial u_1}{\partial x_1}\delta t\right)\cancel{\delta x_1}}$$

$\delta\alpha$

$\to 0$ as $\delta t \to 0$

$$\delta\alpha = \frac{\partial u_2}{\partial x_1}\delta t$$

$$\frac{\Delta\alpha}{\Delta t} = \frac{\partial u_2}{\partial x_1}$$

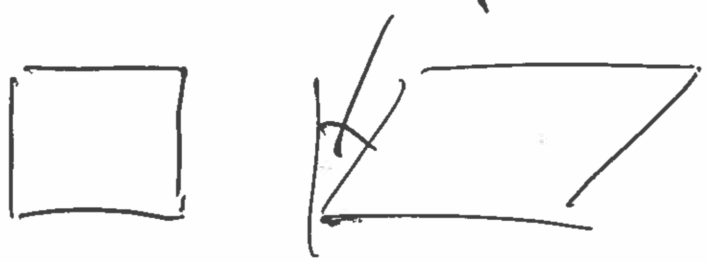
rate of change of angle α experienced by material line

Similarly

$$\frac{D\beta}{Dt} = \frac{\partial u_1}{\partial x_2} \quad \text{(EXERCISE)}$$

Now consider the "shear rate" i.e. the rate of which the angle between these lines changes:

$$\delta\gamma = \delta\alpha + \delta\beta$$

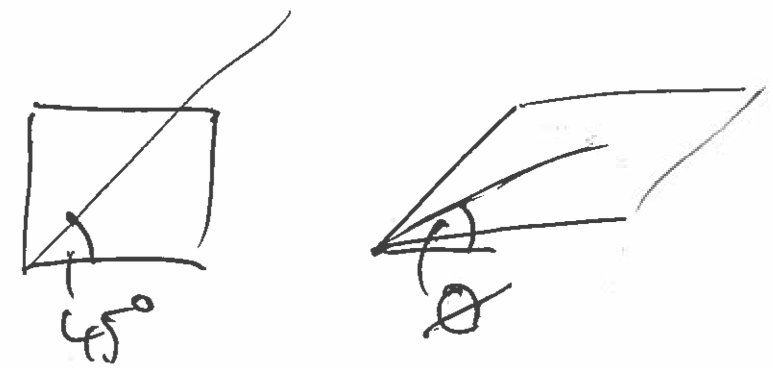


$$\frac{D\gamma}{Dt} = \frac{D\alpha}{Dt} + \frac{D\beta}{Dt} = \left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right)$$

$$= 2\,\varepsilon_{12}$$

The off-diagonal entries in the rate of strain tensor ε_{ij} represent <u>half</u> the shear rate in the plane spanned by x_i & x_j.

Also: The change in angle of the main diagonal in the fluid rectangle is



$$\cancel{\delta\theta = \frac{1}{2}\frac{D\alpha}{Dt} - D}$$

$$\delta\theta = \frac{1}{2}(\delta\alpha - \delta\beta)$$

$$\frac{D\theta}{Dt} = \frac{1}{2}\left(\frac{D\alpha}{Dt} - \frac{D\beta}{Dt}\right)$$

$$= \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right) = \omega_{21} = \omega_3$$

from rate of rotation tensor

$$\underline{\omega} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

Summary:

Motion of fluid in the vicinity of a spatial point can be decomposed into

$$\underline{u}(\underline{x}+\delta\underline{x}) = \underline{u}(\underline{x}) + \underline{\Omega}\times\delta\underline{x} + \underset{\sim}{E}\,\delta\underline{x}$$

translation rotation

rigid body

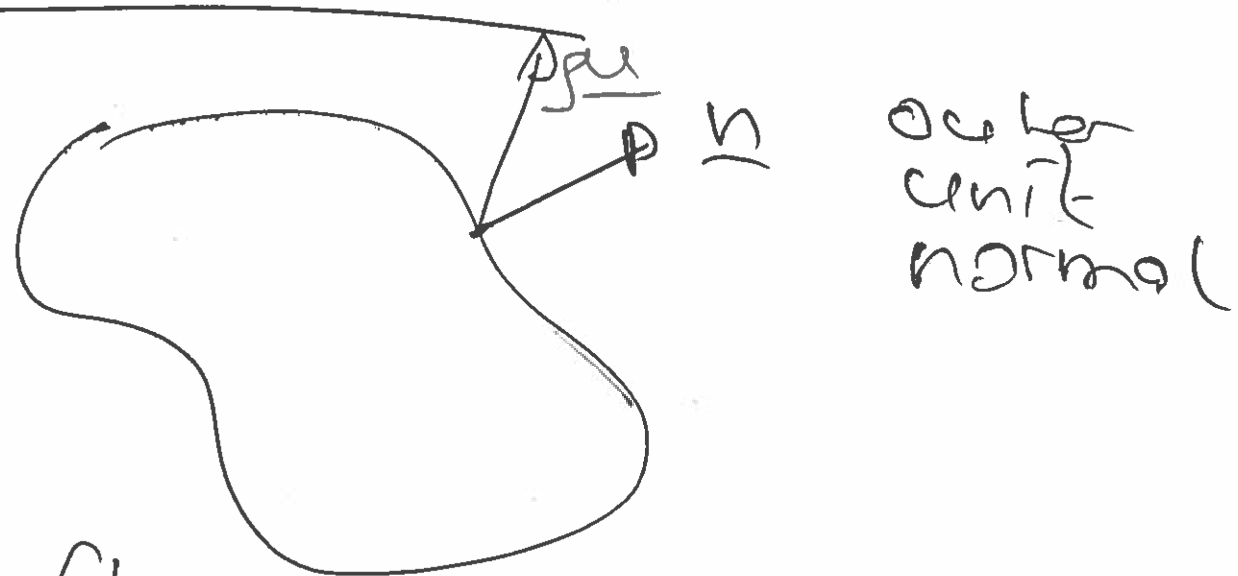
shearing & dilation

$$u_i(x_j+\delta x_j) = u_i(x_j) + \omega_{ij}\,\delta x_j + \varepsilon_{ij}\,\delta x_j$$

Eqn. of continuity

Physics: Mass flux into a spatially fixed (control) volume is equal to the rate of change of mass in that volume.

Integral form:



Mass flux:
density $\rho\left[\frac{kg}{m^3}\right]$ x velocity normal to boundary $\left[\frac{m}{sec}\right]$ x surface area $[m^2]$

$$-\oint \rho \underline{u} \cdot \underline{n} \, dA = \int \frac{\partial \rho}{\partial t} dV$$

$$\int \frac{\partial \rho}{\partial t} dV + \oint (\rho \underline{u}) \cdot \underline{n} \, dA = 0$$

$$\Big\downarrow \text{ divergence theorem}$$

$$\int \frac{\partial \rho}{\partial t} dV + \int \nabla \cdot (\rho \underline{u}) dV = 0$$

$$\int \left(\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \underline{u} \right) dV = 0$$

if this has to hold for all control volumes the integrand has to vanish everywhere:

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \underline{u} = 0}$$