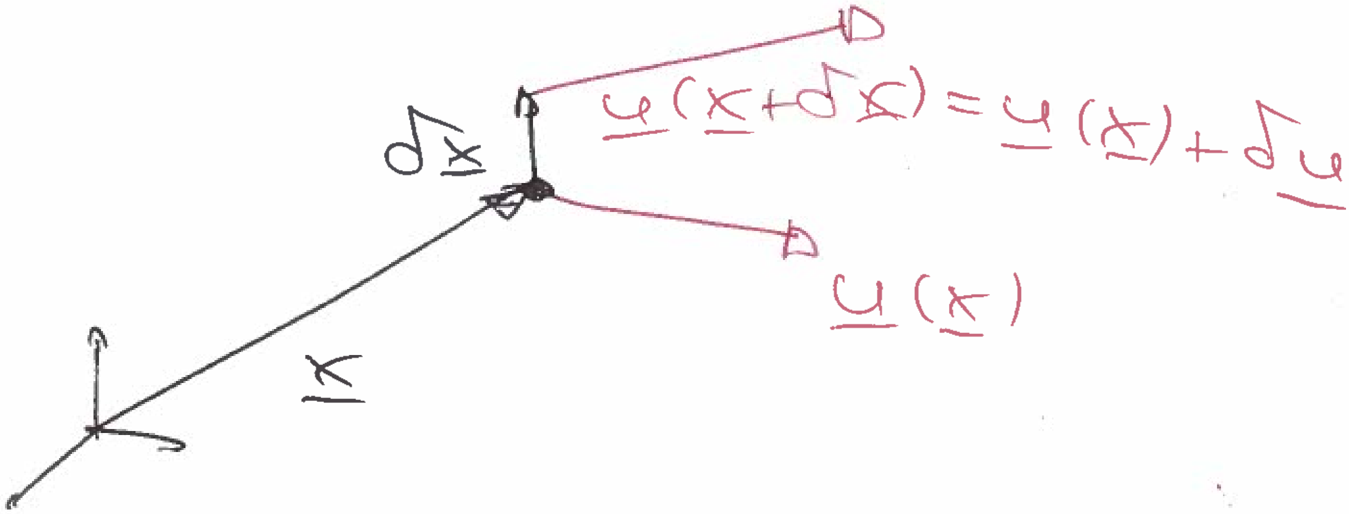
(1



$$\delta u_i = \frac{\partial u_i}{\partial x_j}\,\delta x_j$$

$\frac{\partial u_i}{\partial x_j} = 0 \Rightarrow$ pure translation

$\frac{\partial u_i}{\partial x_j}$ contains all other "modes"

To see this, split $\frac{\partial u_i}{\partial x_j}$ into sym. & anti-sym. parts

$$\frac{\partial u_i}{\partial x_j} = \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}_{\substack{\varepsilon_{ij} = \varepsilon_{ji} \\ \text{rate of strain tensor}}} + \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)}_{\substack{\omega_{ij} = -\omega_{ji} \\ \text{rate of rotation tensor}}}$$

$$\delta u_i = \underbrace{\varepsilon_{ij}\,\delta x_j}_{\substack{\text{change}\\ \text{in veloc.}\\ \text{due to}\\ \text{defromation}}} + \underbrace{\omega_{ij}\,\delta x_j}_{\substack{\text{change in}\\ \text{veloc. due}\\ \text{to rotation}}}$$

① Rigid body rotation / vorticity

Consider δu_i due to ω_{ij}:

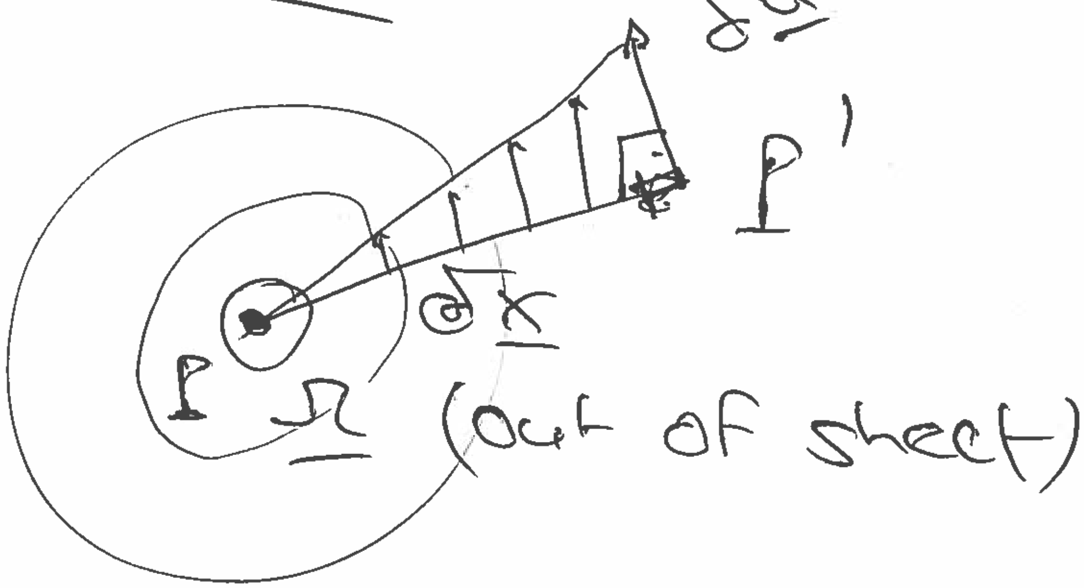
$$\delta u_i = \omega_{ij}\,\delta x_j = -\omega_{ji}\,\delta x_j$$

$$\begin{pmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \end{pmatrix} = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{pmatrix}$$

$$\delta \underline{u} = \underline{\Omega} \times \delta \underline{x}$$

where $\underline{\Omega} = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix}$ is the rate of rotation vector.

Sketch:



$\delta \underline{u} = \underline{\Omega} \times \delta \underline{x}$ is the veloc. induced at P' by a rigid body rotation about P.

$$\underline{\Omega} = \frac{1}{2} \nabla \times \underline{u}$$

$$\underline{\Omega} = \frac{1}{2} \begin{pmatrix} \frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \\ \frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \end{pmatrix} = \frac{1}{2} \underline{\omega}$$

vorticity

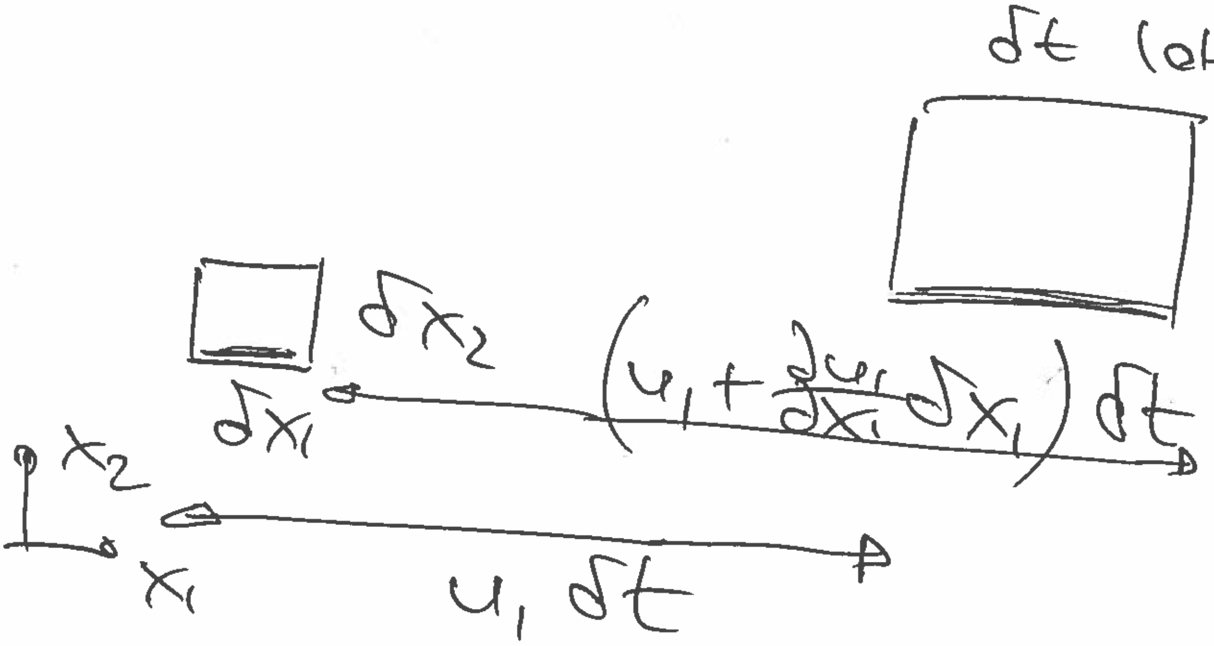
2 The rate of strain

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

"contains" shearing & dilation

(i) Extensional rate of strain

2D sketch:



$$\text{strain} = \frac{\text{length} - \text{old length}}{\text{old length}}$$

$$= \frac{\left\{\left[\cancel{\delta x_1} + \left(\cancel{u_1} + \frac{\partial u_1}{\partial x_1}\cancel{\delta x_1}\right)\delta t\right] - \cancel{u_1 \delta t}\right\} \cancel{- \delta x_1}}{\cancel{\delta x_1}}$$

$$\text{strain} = \frac{\partial u_1}{\partial x_1}\,\delta t$$

$$\text{rate of strain} = \frac{\partial(\text{strain})}{\partial t}$$

$$= \frac{\partial u_1}{\partial x_1} = \varepsilon_{11}$$

(similar for other directions)

ε_{11} (etc) i.e. the diagonal entries of ε_{ij} represent the extensional rate of strain in the direction of the coordinate axes.