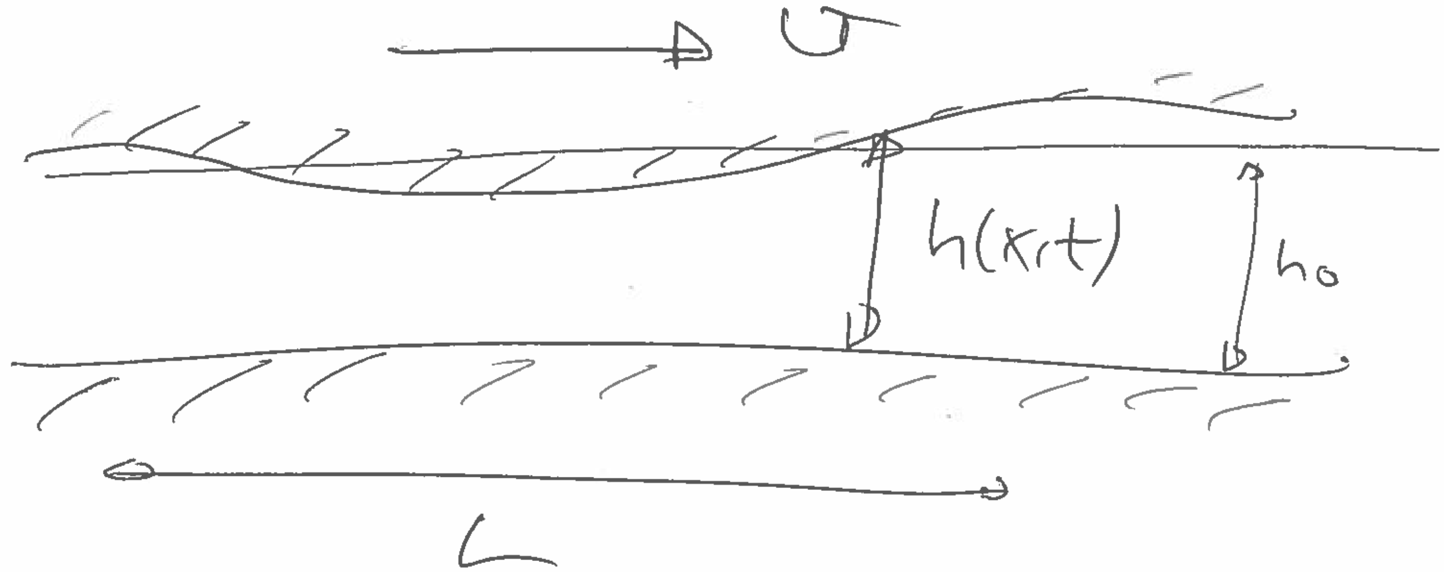
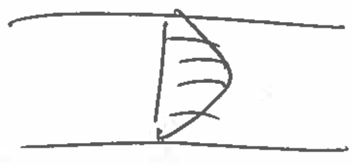
Lubrication theory 2D



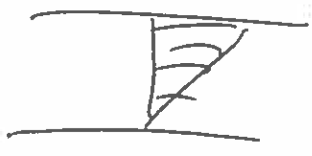
$$\frac{h_0}{L} \ll 1$$

$$\boxed{\begin{aligned} 0 &= -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \\ 0 &= -\frac{\partial p}{\partial y} \end{aligned}}$$

$$u(x,y,t) = \underbrace{\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2 - h(x,t)\,y\right)} + \underbrace{U\frac{y}{h(x,t)}} \quad (*)$$



press. driven



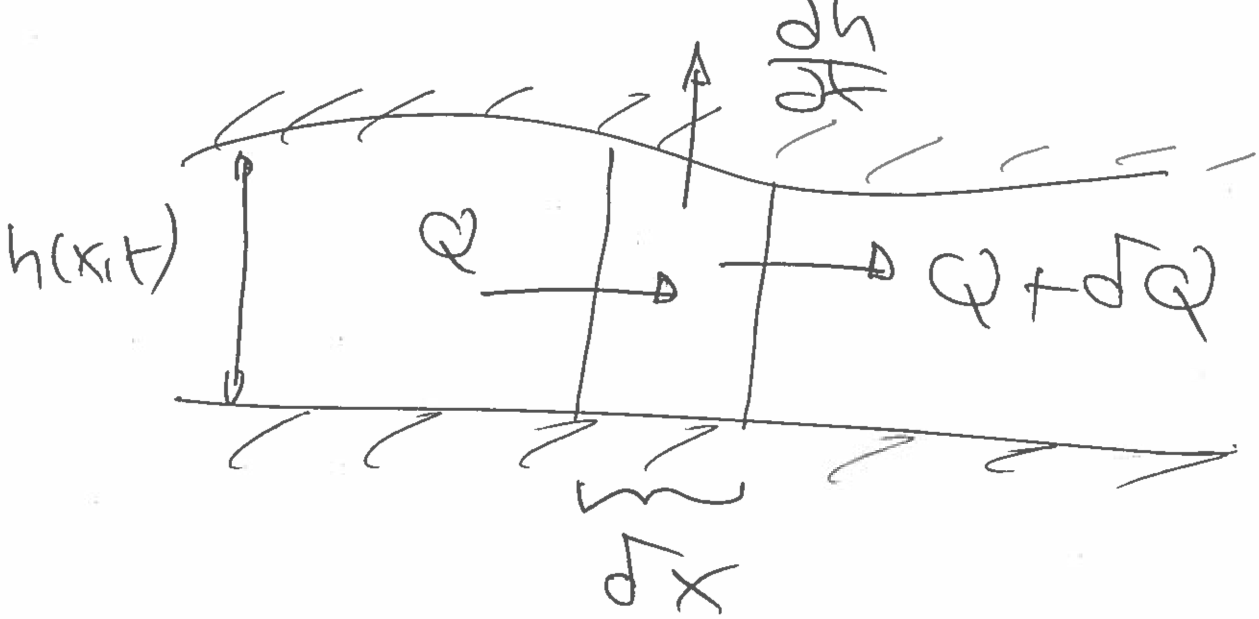
shear

Also: what is $\frac{\partial p}{\partial x}$?

Additional requirement: volume conservation!

Consider volume flux through cross-section of channel:

$$Q(x,t) = \int_0^{h(x,t)} u(x,y,t)\,dy$$



Net outflow

$$\cancel{Q} + \delta Q + \frac{\partial h}{\partial t}\,\delta x - \cancel{Q} = 0$$

$$\frac{\delta Q}{\delta x} = -\frac{\partial h}{\partial t}$$ now $\delta x \to 0$

$$\frac{\partial Q}{\partial x} = -\frac{\partial h}{\partial t}$$

$$Q = \int_0^h u \, dy$$

from (*) & integrate

$\vdots$

$$Q = -\frac{1}{12\mu}\frac{\partial p}{\partial x} h^3 + \frac{1}{2} U h$$

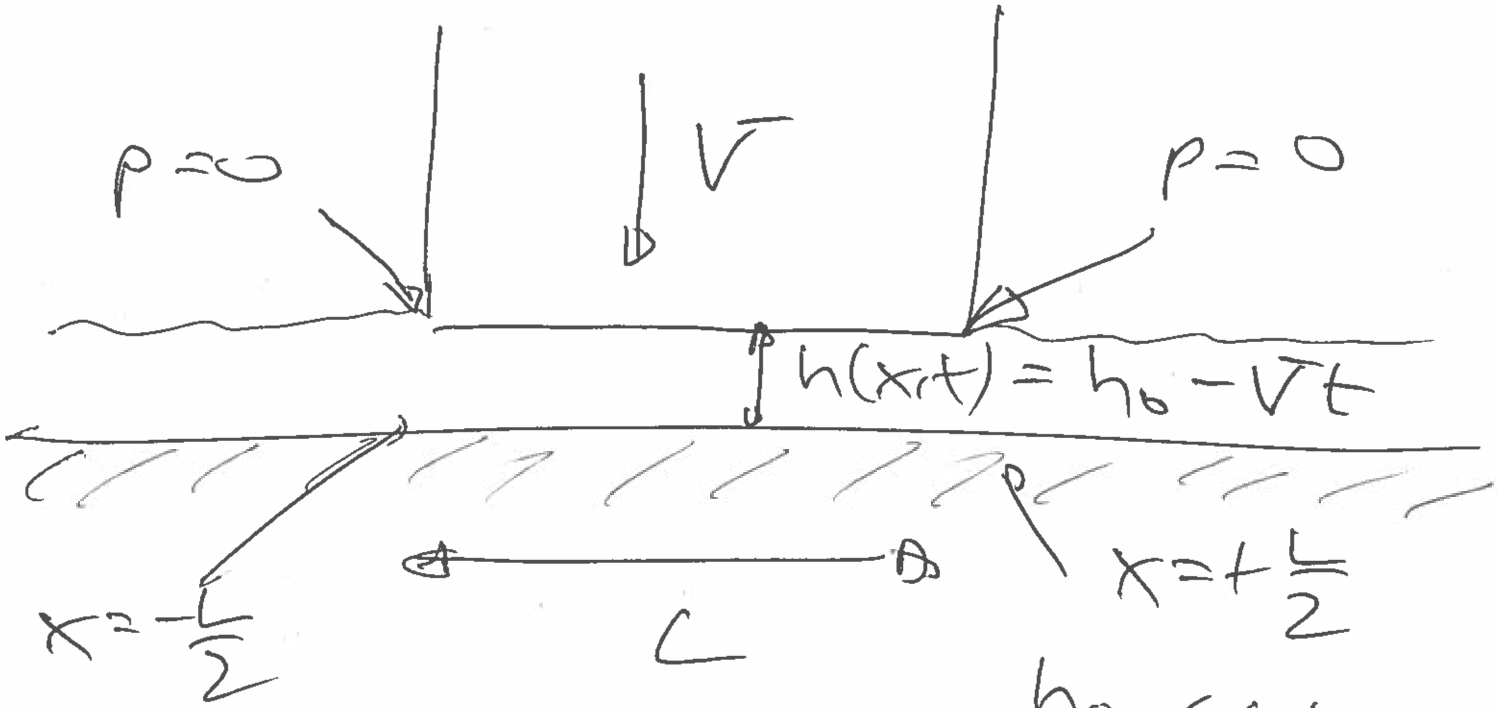
$$\frac{\partial Q}{\partial x} = -\frac{1}{12\mu}\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{1}{2} U \frac{\partial h}{\partial x} = -\frac{\partial h}{\partial t}$$

$$\boxed{\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) = 12 \frac{\partial h}{\partial t} + 6 U \frac{\partial h}{\partial x}}$$

$h(x,t)$ given

This is the Reynolds' lubrication equation.
Given $h(x,t)$, U this is an eqn. for $p(x,t)$.
$u(x,y,t)$ follows from (*).

Example: Squeeze film:



$$\frac{h_0}{L} \ll 1$$

Here: $\frac{\partial h}{\partial x} = 0 \qquad \frac{\partial h}{\partial t} = \dot{h} = -V$

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) = 12\dot{h}$$

$$\frac{h^3}{\mu}\frac{\partial p}{\partial x} = 12\dot{h}x + \tilde{A}(t)$$

$$\frac{\partial p}{\partial x} = \frac{12\dot{h}\mu}{h^3}x + A(t)$$

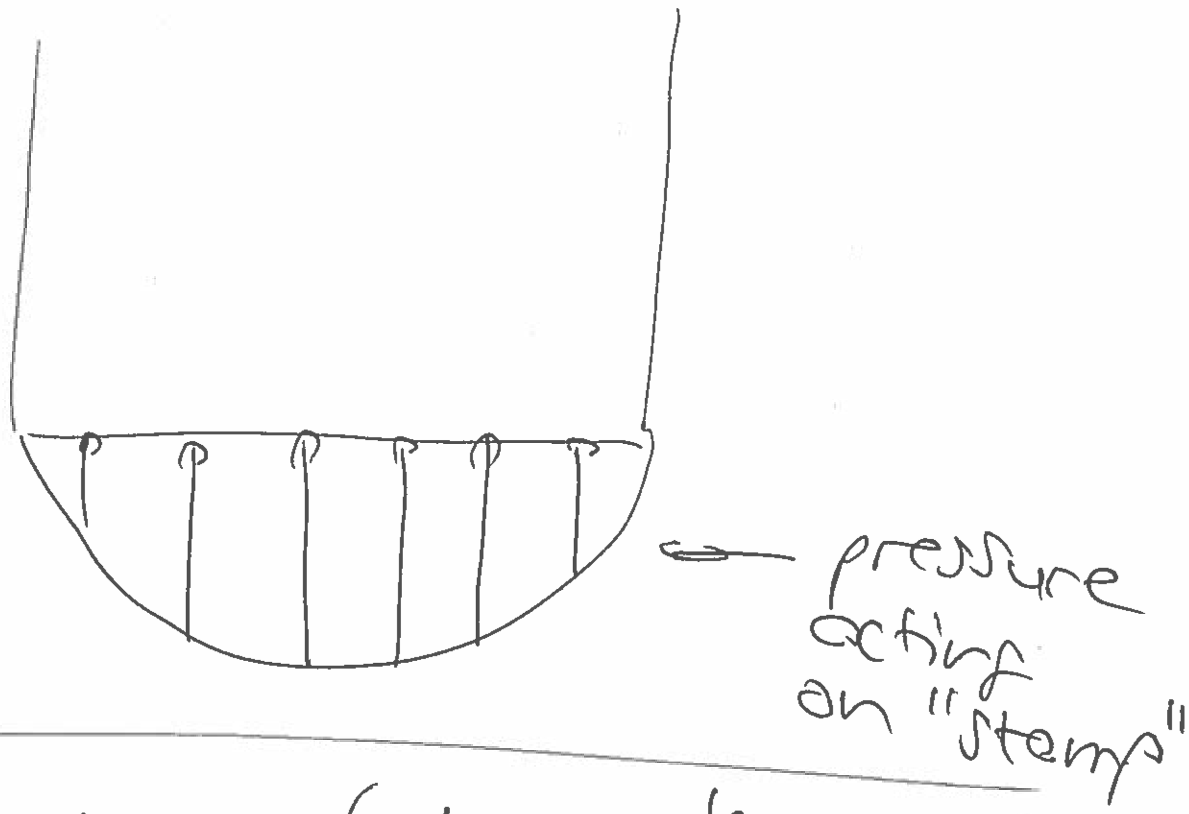
$$p(x,t) = \frac{6\dot{h}\mu}{h^3}x^2 + A(t)x + B(t)$$

BC: $p(x=-L/2) = p(x=L/2) = 0$

$$\boxed{p(x,t) = \frac{6\dot{h}\mu}{h^3}\left(x^2 - \left(\frac{L}{2}\right)^2\right)}$$

$$h(x,t) = h_0 - Vt$$

$$\dot{h} = -V$$



As $h \to 0$ $\left(t \to \frac{h_0}{V}\right)$ the pressure becomes ∞.

EXAM

- kinem. ε_{ij} ω_{ij} interpret.
  $\frac{D\cdot}{Dt}$
- BC, IC, $t_i = \tau_{ij} n_j$
  $\underline{u} = \cdot$
- ∥ flow, deriv, apply
- nondim, scaling
- MUST KNOW:
  Eqns in cartesian coords
  N-St. $\nabla^4 \psi = 0$ $\psi \to \underline{u}$
  ω_{ij} ε_{ij}
  $\tau_{ij} = -p\delta_{ij} + 2\mu\varepsilon_{ij}$
- No cubic!