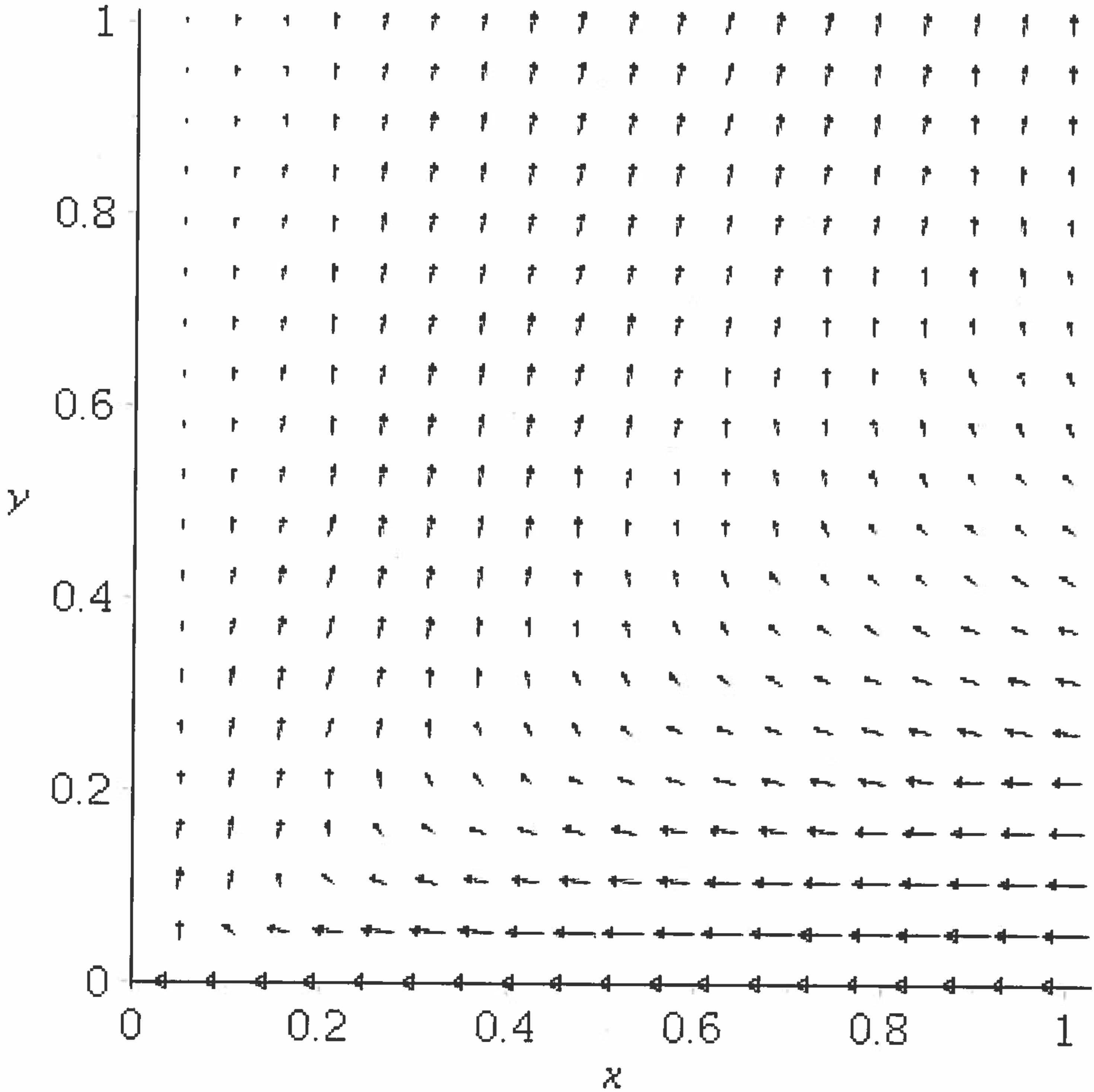
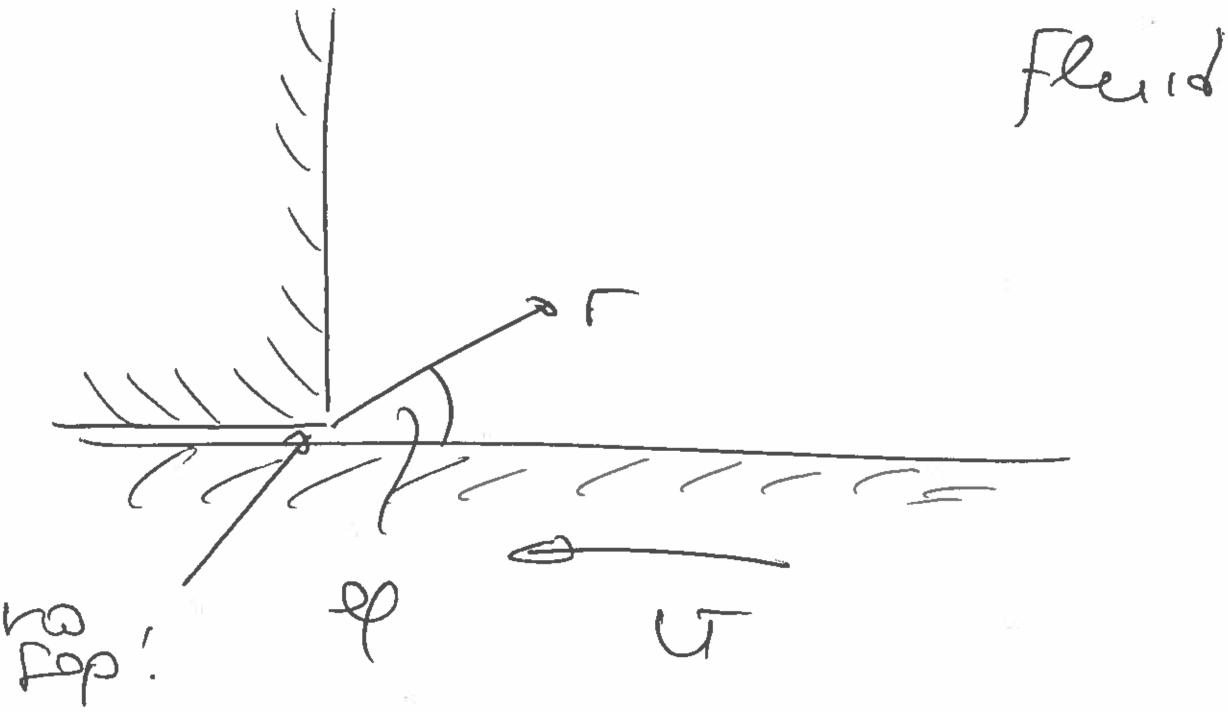
(D)





slow, viscous $Re = 0$

$$\nabla^4 \psi = 0 \quad + \text{BC}$$

$$\psi = \frac{Ur}{\left(\frac{\pi}{2}\right)^2 - 1}\left(-\left(\frac{\pi}{2}\right)^2 \sin\varphi + \varphi\cos\varphi + \frac{\pi}{2}\varphi\sin\varphi\right)$$

$\underline{u}$ indep. of r!

Discussion

(I) Non-uniformity of soln.

Had assumed

$$Re = \frac{UL\rho}{\mu} \ll 1$$

μ, ρ given

$\mathcal{U} = U$

$L = ??!$

The problem has no intrinsic length scale!

Choose one!

E.g. distance from corner.

For given fluid (μ, ρ) & veloc. U I can choose L small enough to make Re small.

Stokes eqns & hence our solution becomes more & more accurate the closer to the corner we are.

However at large distances from the corner the full N. St. have to be solved.

$\Rightarrow$ Nonuniformity of soln.

(II) Force on bottom wall

tangential component of traction is $\tau_{r\varphi}$ [Exercise]

$$F = \left| \int_{r=0}^{\infty} \tau_{r\varphi} \, dr \right|$$

$$\tau_{ij} = -p\,\delta_{ij} + 2\mu\,\varepsilon_{ij}$$

$i,j = (r,\varphi,z)$

$$\tau_{r\varphi} = 2\mu\,\varepsilon_{r\varphi}$$

from handout

$$\frac{\tau_{r\varphi}}{\mu} = r\frac{\partial}{\partial r}\left(\frac{u}{r}\right) + \frac{1}{r}\frac{\partial u}{\partial \varphi}$$

of $\varphi = 0$

Had noticed $\underline{u}$ is indep. of r.

$$r\frac{\partial}{\partial r}\left(\frac{u}{r}\right) \sim r^{-1}$$

$$\frac{1}{r}\frac{\partial u}{\partial \varphi} \sim r^{-1}$$

$$\tau_{r\varphi} \sim r^{-1}$$

$$F = \mu \ldots \int_{r=0}^{\infty} \frac{1}{r}\,dr = \infty$$

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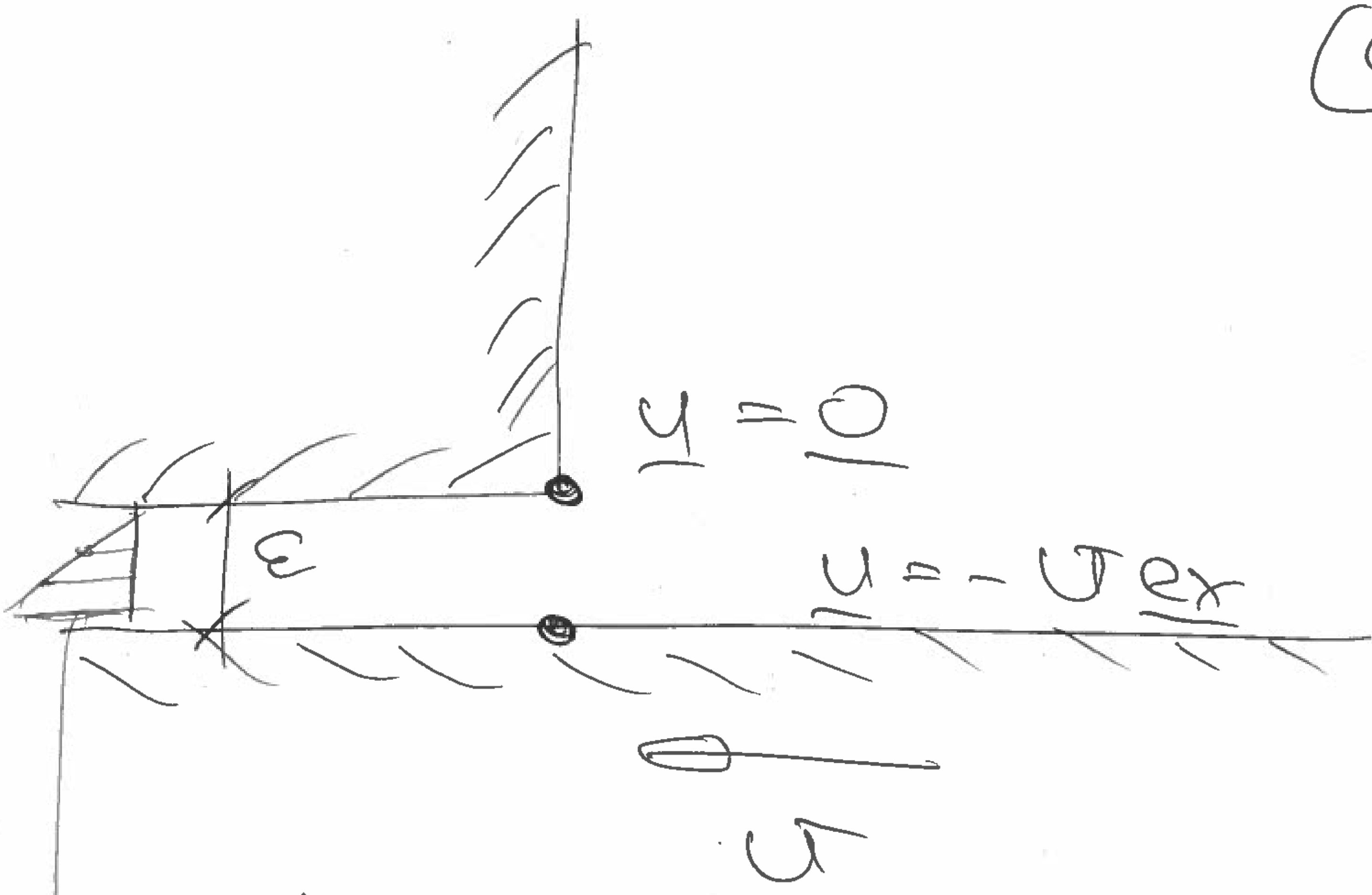
Horizontal force needed to move the plate is infinite.

Maybe OK because plate is ∞ long.

Compute force on plate between $\cancel{r}=0$ & $r=L$

$$F_L = \mu \ldots \int_{r=0}^{L} \frac{1}{r} \, dr = \infty$$

So a second source for the unboundedness of the force is associated with the corner.



we have considered $\varepsilon = 0$

shear rate $\quad \varepsilon_{xy} \approx \dfrac{U}{\varepsilon}$

as $\varepsilon \to 0 \quad \varepsilon_{xy} \sim \tau_{xy} \to \infty$
in a non-integrable way.