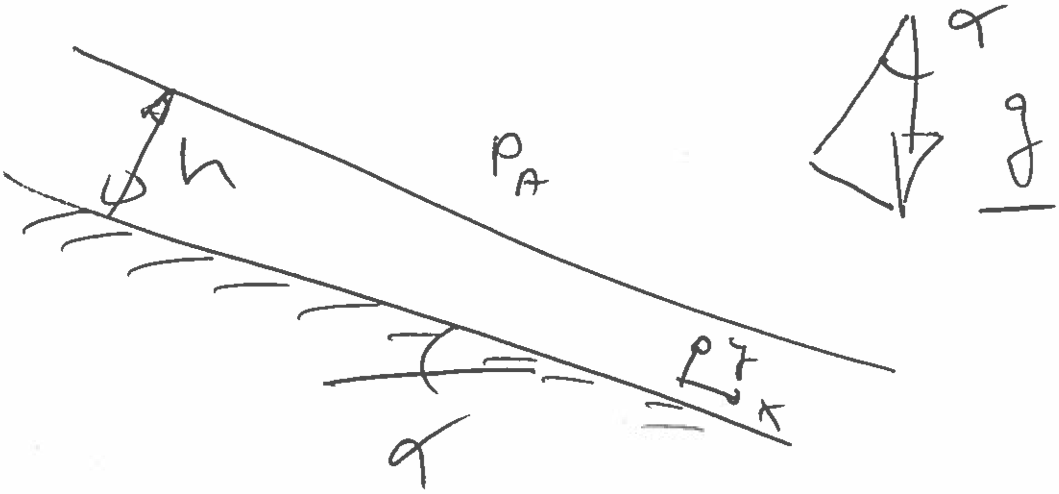


$$\boxed{0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \rho g \sin\alpha} \quad (1)$$

$$\boxed{0 = -\frac{\partial p}{\partial y} - \rho g \cos\alpha} \quad (2)$$

No slip: $\boxed{u = 0 \quad @\ y = 0}$ (3)

Traction BC $\boxed{p(y=h) = p_A}$ (4)

$$\boxed{0 = \mu \frac{\partial u}{\partial y}} \ @\ y = h \quad (5)$$

(2) & (4):

$$p(x,y) = p_A + \rho g \cos\alpha \, (h-y)$$

into (1)

$$0 = \cancel{-\frac{\partial p}{\partial x}} + \mu \frac{\partial^2 u}{\partial y^2} + \rho g \sin\alpha$$

Flow is driven only by gravity.

$$\frac{\partial^2 u}{\partial y^2} = \frac{d^2 u}{dy^2} \quad \text{because } u(y):$$

$$\frac{d^2 u}{dy^2} = -\frac{\rho g \sin\alpha}{\mu}$$

integrate twice:

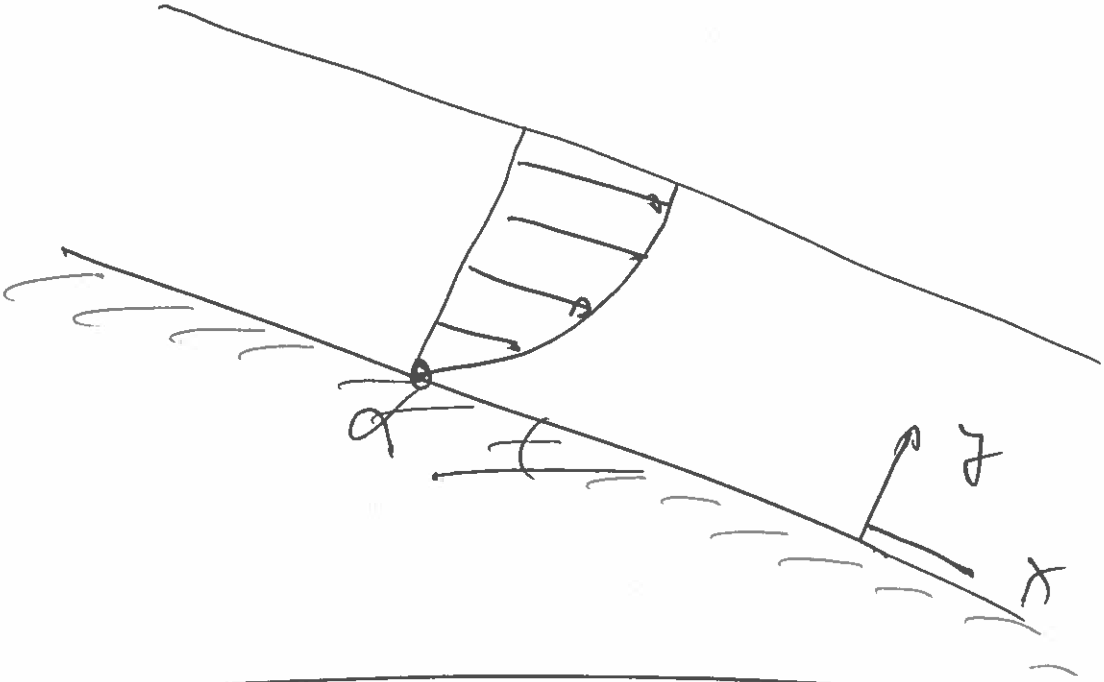
$$u(y) = -\frac{1}{2}\frac{g \sin\alpha}{\nu} y^2 + Ay + B$$

A & B from BCs:

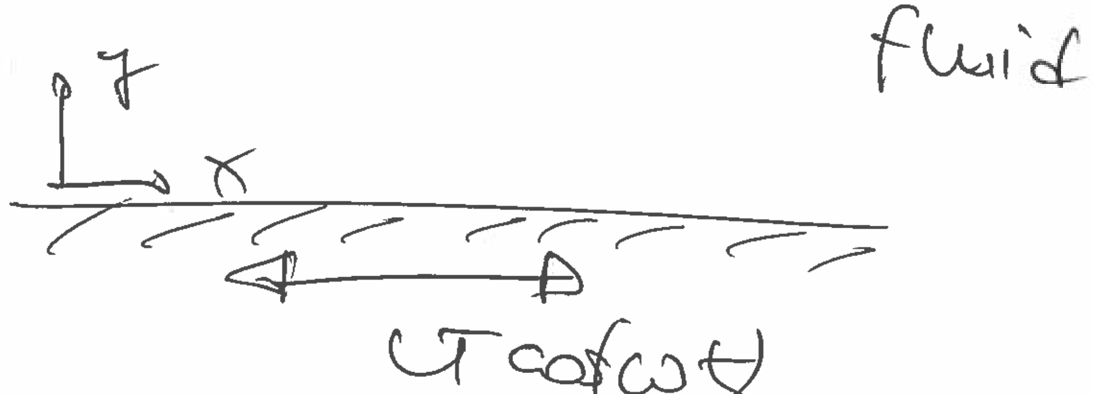
$$u(y=0) = 0$$

$$\mu \left.\frac{du}{dy}\right|_{y=h} = 0$$

$$\boxed{u(y) = \frac{g \sin\alpha}{\nu}\left(hy - \frac{1}{2}y^2\right)}$$



Example: The vibrating plate



- No body force
- Assume: no press. gradient

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \qquad u(y,t)$$

BC:

$u = U\cos(\omega t)$ at $y = 0$

$u \to 0$ as $y \to \infty$

Look for time-periodic solutions:

$$u(y,t) = f(y)\cos(\omega t + \phi(y))$$

Easier to use complex variables

$$u(y,t) = f(y)\, e^{i\omega t}$$

then take real part.

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$

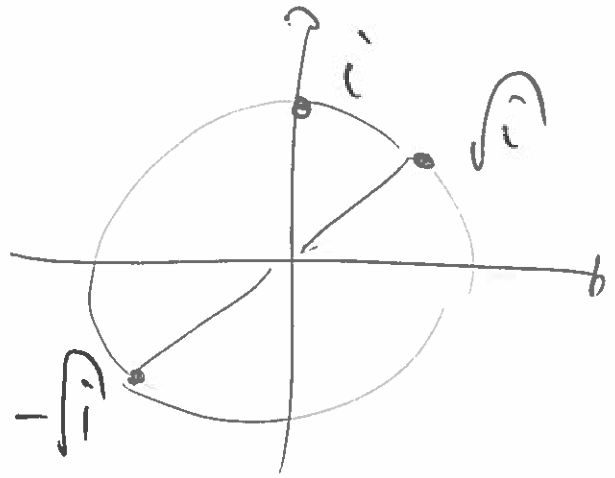
$$i\omega f = \nu f''$$

$$\boxed{f'' - \frac{i\omega}{\nu} f = 0}$$

$$f \sim e^{\lambda y}$$

$$\lambda^2 - \frac{i\omega}{\nu} = 0$$

$$\lambda = \pm\sqrt{\frac{i\omega}{\nu}} = \pm\sqrt{i}\sqrt{\frac{\omega}{\nu}}$$



$$\lambda = \pm \frac{1}{2}\sqrt{2}\,(1+i)\sqrt{\frac{\omega}{\nu}}$$

$$\lambda = \pm (1+i)\sqrt{\frac{\omega}{2\nu}}$$

$$f(y) = A\, e^{(1+i)\sqrt{\frac{\omega}{2\nu}}\, y} + B\, e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}\, y}$$

BC: $\cancel{u(y=0)=0}$

$$\Rightarrow \cancel{f(0) = \boxed{0 = A+B}}$$

$$f \to 0 \text{ as } y \to \infty : \quad A = 0$$

$$u(y=0,t) = U e^{i\omega t}$$

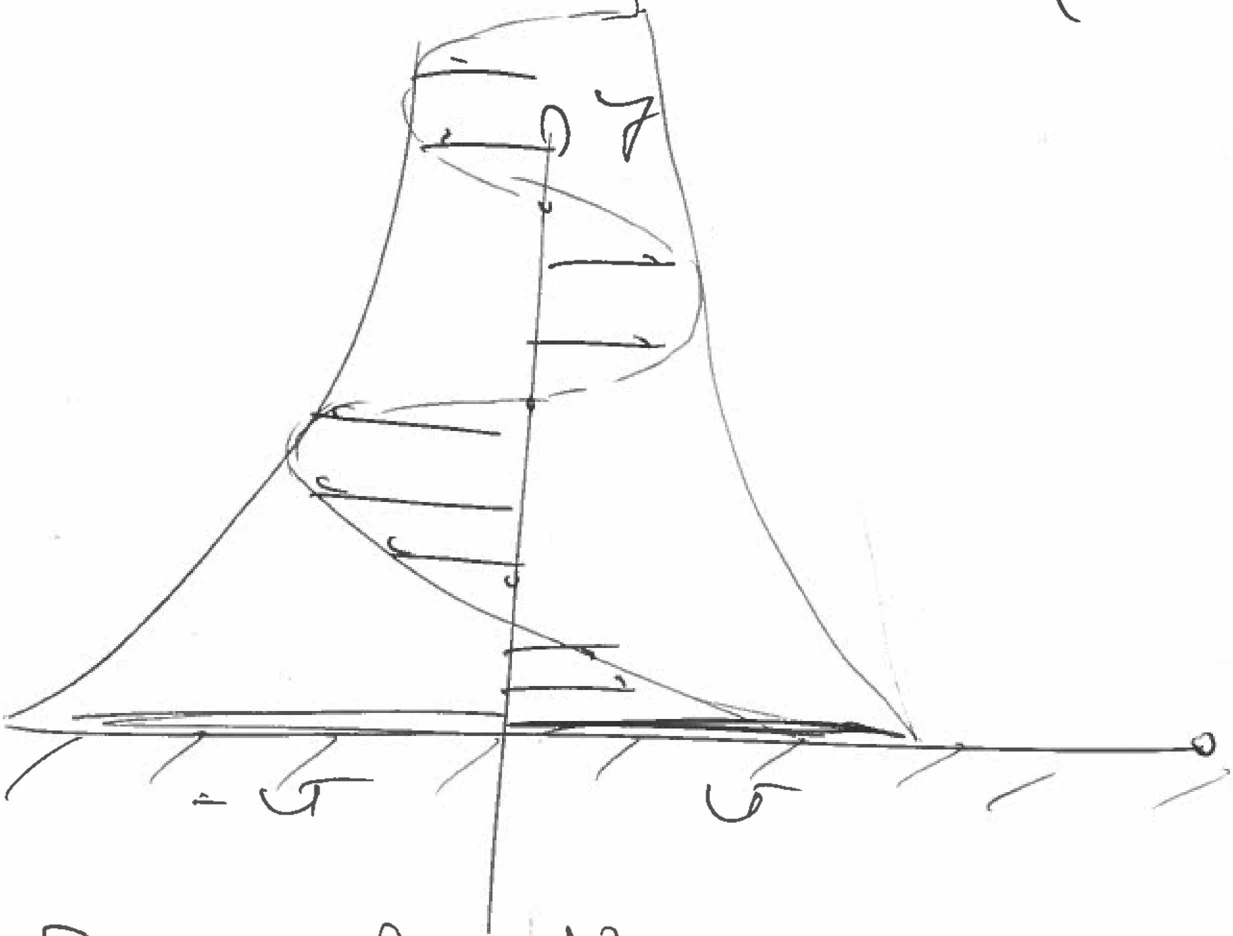
$$f(y=0)\, e^{i\omega t} = U e^{i\omega t}$$

$$f(y=0) = \boxed{U = A+B}$$

$$u(y,t) = U e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}\, y}\, e^{i\omega t}$$

extract real part:

$$u(y,t) = U e^{-\sqrt{\frac{\omega}{2\nu}}\, y} \cos\left(\omega t - \sqrt{\frac{\omega}{2\nu}}\, y\right)$$



See animation on www.