# MATH35001: EXAMPLE SHEET[1] II

**1.)** A 2D flow field is given by $u_1 = ax_2$ and $u_2 = ax_1$, where $a > 0$ is a constant.

**a)** Sketch the flow field (begin by considering the velocity vectors on the coordinate axes and on the main diagonals).

**b)** Determine the trajectories $x_i^p(t)$ of particles which are at position $x_i = X_i$ at time $t = 0$ by integrating the equations of motion $dx_i^p(t)/dt = u_i(x_j^p(t), t)$.

**c)** Determine the acceleration of material particles directly from their trajectories (i.e. by evaluating $a_i = d^2x_i^p(t)/dt^2$).

**d)** Compare this to the result obtained by using the material derivative $a_i = Du_i/Dt$.

**e)** What is the divergence of the flow field?

**2.)** A 2D flow field is given by $u_1 = Ux_2$ and $u_2 = 0$, where $U > 0$ is a known constant.

**a)** Sketch the flow field in the region $x_2 \in [0, 1]$.

**b)** Determine the rate of strain and the rate of rotation tensors $\epsilon_{ij}$ and $\omega_{ij}$.

**c)** Sketch the deformation of a small rectangular fluid element whose edges of lengths $\delta x_1$ and $\delta x_2$ are parallel to the $x_1$ and $x_2$ axes, respectively. [**Hint**: Sketch the corners of the fluid element at time $t$ and at time $t + \delta t$.]

Relate this to the interpretation of the rate of strain and rate of rotation tensors. [**Hint**: To interpret the entries in the rate of rotation tensor, consider the average of the rate at which the lines that were initially parallel to the $x_1$ and $x_2$ axes rotate about the $x_3$-axis.]

*Coursework*

Please exchange your solution to question 2 with your "marking buddy" and assess each other's work, using the master solution made available on the course webpage (probably in week 3).

[1] Any feedback to: *M.Heil@maths.manchester.ac.uk*