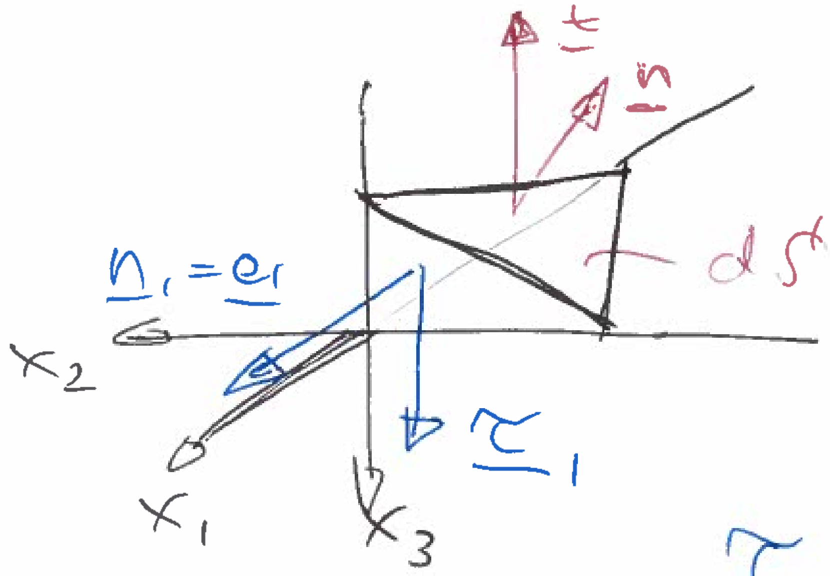


$$\underline{t} = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix}$$

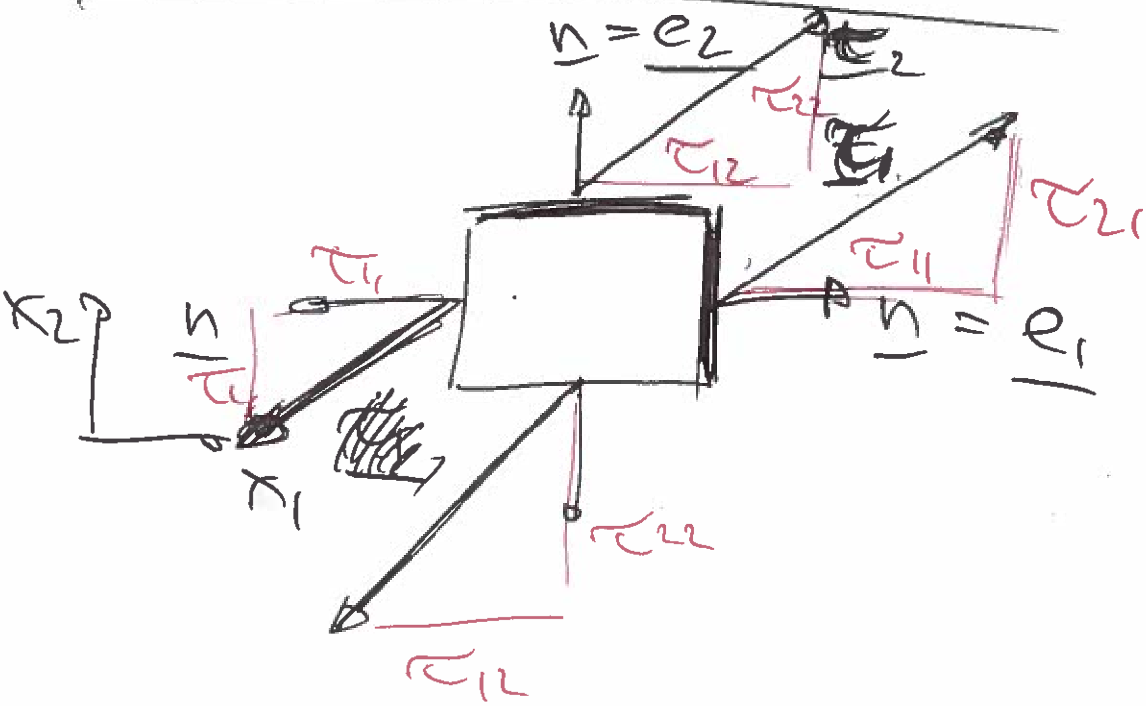
$$\underline{\tau}_1 = \begin{pmatrix} \tau_{11} \\ \tau_{21} \\ \tau_{31} \end{pmatrix}$$

$$\boxed{t_i = \tau_{ij} n_j}$$

τ_{ij} = stress tensor

τ_{ij} represents: the traction/stress component in the positive x_i-direction on the face x_j = const whose normal points in the positive x_j-direction.

Illustration (2D)



Using the stress tensor we can determine the stress at any point in the fluid.

Later: Determine τ_{ij} as a fct. of the flow field.

Particular stress states

(i) Hydrostatic pressure

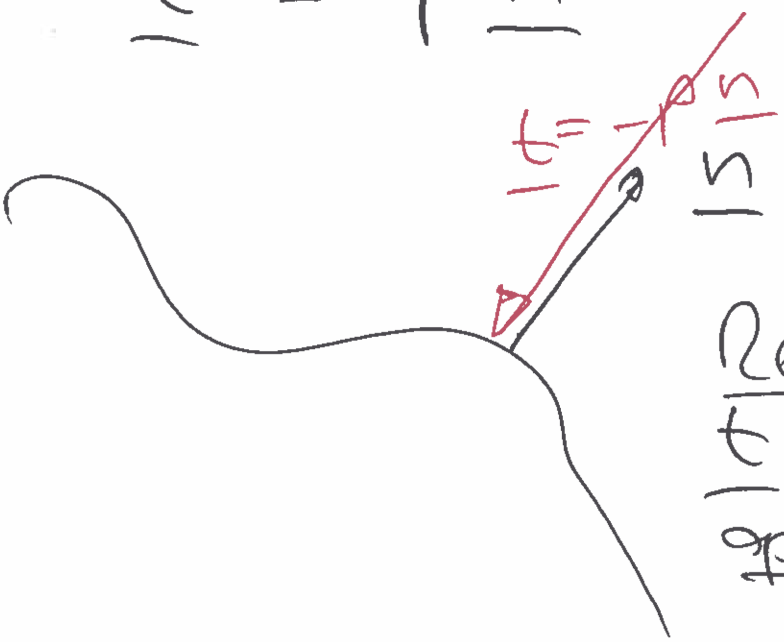
$$\tau_{ij} = -p\,\delta_{ij}$$

implies that the traction / stress is always normal to the plane it acts on & uniform in all directions.

$$t_i = \tau_{ij} n_j = -p \underbrace{\delta_{ij} n_j}$$

$$t_i = -p n_i$$

$$\underline{t} = -p\,\underline{n}$$

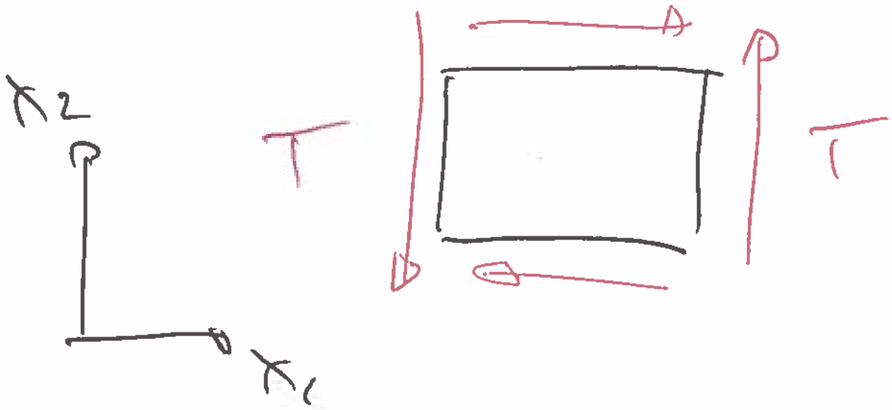


Remember:
$\underline{t}$ is the applied traction

(ii) pure shear stress

E.g.:

$$\left.\begin{aligned} \tau_{12} &= \tau_{21} = T \\ \tau_{11} &= \tau_{22} = 0 \end{aligned}\right) \quad 2D$$

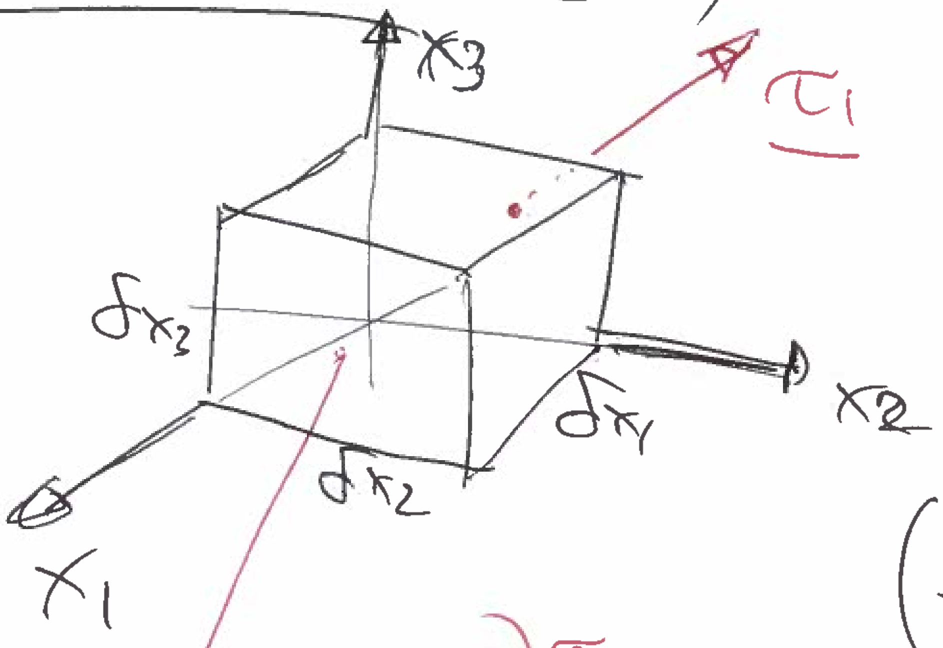


③ Equilibrium of forces

Cauchy's eqn

"$\sum$ forces = mass $\times$ accel"
for a blob of fluid

Sketch: (3P)



$$\underline{\tau}_1 + \frac{\partial \underline{\tau}_1}{\partial x_1} \delta x_1$$

(Similar on other faces)

Tractions act on the same areas of the front & back $\Rightarrow$ only the increments contribute to the net force on the blob.

$$\left(\frac{\partial \tau_{1}}{\partial x_{1}}\cancel{\delta x_1}\right)\cancel{\delta x_2\,\delta x_3} +$$

$$+\left(\frac{\partial \tau_{2}}{\partial x_{2}}\cancel{\delta x_2}\right)\cancel{\delta x_1\,\delta x_3} +$$

$$+\left(\frac{\partial \tau_{3}}{\partial x_{3}}\cancel{\delta x_3}\right)\cancel{\delta x_1\,\delta x_2}$$

$$+\ \rho\,\underline{F}\ \cancel{\delta x_1\,\delta x_2\,\delta x_3}$$

↳ body force per unit mass

$$= \rho\,\cancel{\delta x_1\,\delta x_2\,\delta x_3}\ \frac{D\underline{u}}{Dt}$$

(Here $\underline{F}$ is a body force e.g. gravity, magnetic forces etc.)

In components:

$$\boxed{\frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i = \rho \frac{D u_i}{Dt}}$$

Cauchy's eqn!

$$\boxed{\frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i = \rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right)}$$

4 Symmetry of the stress tensor

$$\tau_{ij} = \tau_{ji}$$