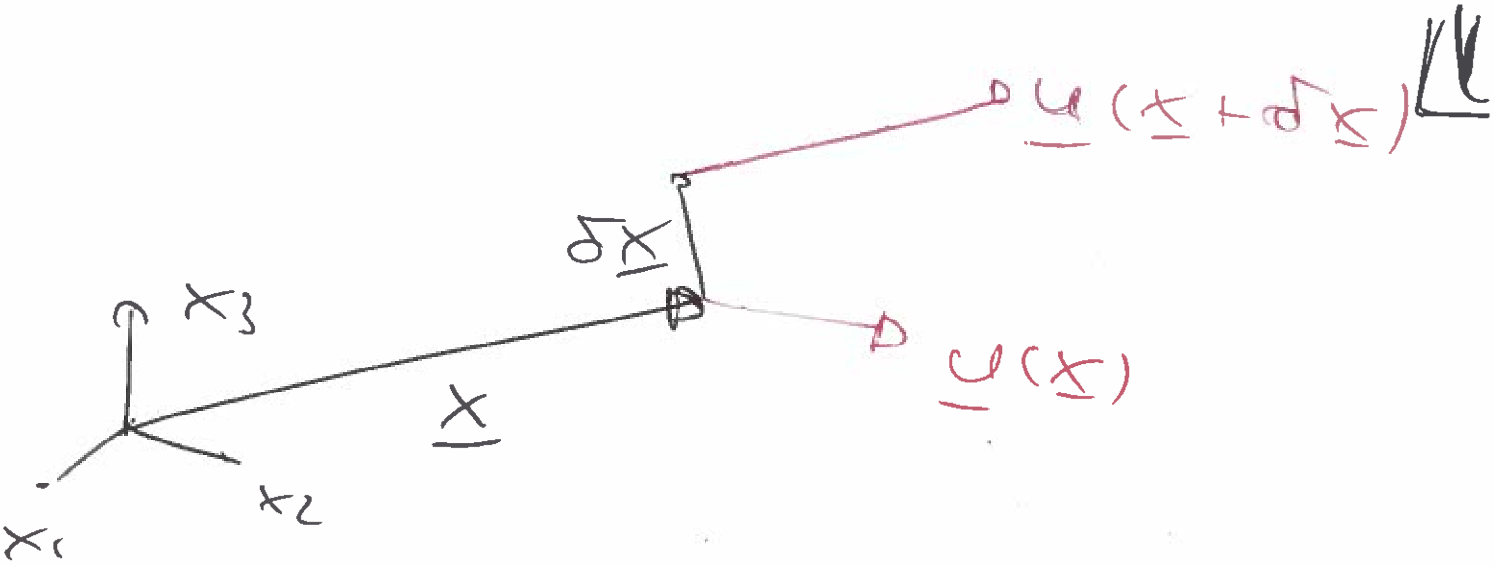


$$\delta\underline{u} = \underline{u}(\underline{x}+\delta\underline{x}) - \underline{u}(\underline{x})$$

$$\delta u_i = 0 + \frac{\partial u_i}{\partial x_j}\delta x_j$$

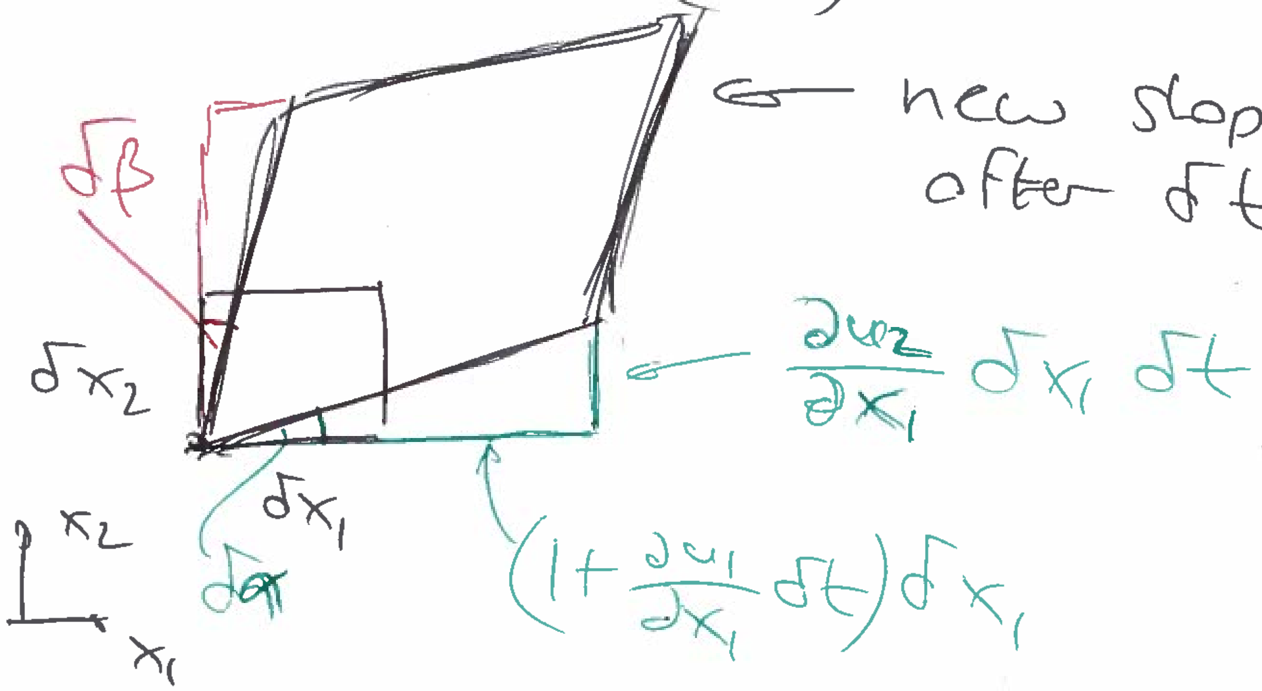
$$= 0 + \omega_{ij}\,\delta x_j + \varepsilon_{ij}\,\delta x_j$$

translation rotation dilation & shearing

$$\left(1+\frac{\partial u_1}{\partial x_1}\delta t\right)\delta x_1$$

(ii) Shear rate of strain

Illustration (2D)



$$\tan \delta\alpha = \frac{\frac{\partial u_2}{\partial x_1}\cancel{\delta x_1}\,\delta t}{\left(1+\frac{\partial u_1}{\partial x_1}\delta t\right)\cancel{\delta x_1}}$$

Now $\delta t \rightarrow 0$; $\delta\alpha \rightarrow 0$:

$$\tan \delta\alpha \rightarrow \delta\alpha$$

$$\delta\alpha = \frac{\partial u_2}{\partial x_1}\delta t$$

$$\frac{\delta\alpha}{\delta t} = \frac{\partial u_2}{\partial x_1}$$

$$\boxed{\frac{D\alpha}{Dt} = \frac{\partial u_2}{\partial x_1}}$$

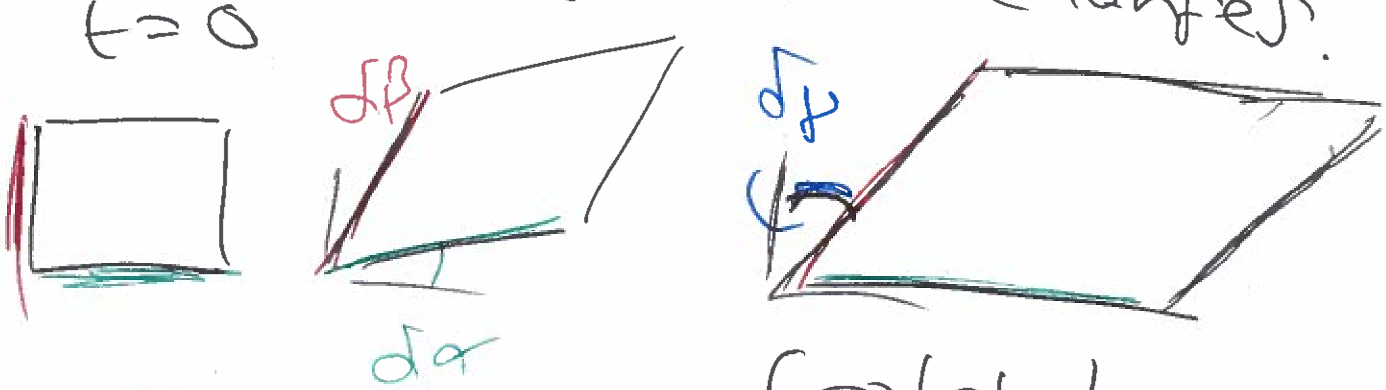
rate of change of angle against horizontal

Similarly:

$$\boxed{\frac{D\beta}{Dt} = \frac{\partial u_1}{\partial x_2}} \quad \text{(Exercise)}$$

Now consider "shear rate"

= the rate at which the initially right angle between material lines parallel to the x_1 & x_2 axes changes.



(rotate)

$$\delta\gamma = \delta\alpha + \delta\beta$$

$$\frac{D\gamma}{Dt} = \frac{D\alpha}{Dt} + \frac{D\beta}{Dt} = \left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right)$$

$$= 2\,\varepsilon_{12}$$

The off-diagonal entries of ε_{ij} represent half the shear rate in the plane spanned by x_i & x_j.

Summary:

Motion of fluid in the vicinity of a fixed spatial point can be decomposed into:

$$\underline{u}(\underline{x}+\delta\underline{x}) = \underline{u}(\underline{x}) + \underline{\Omega}\times\delta\underline{x} + \underset{\sim}{E}\,\delta\underline{x}$$

translation rotation

rigid body motions

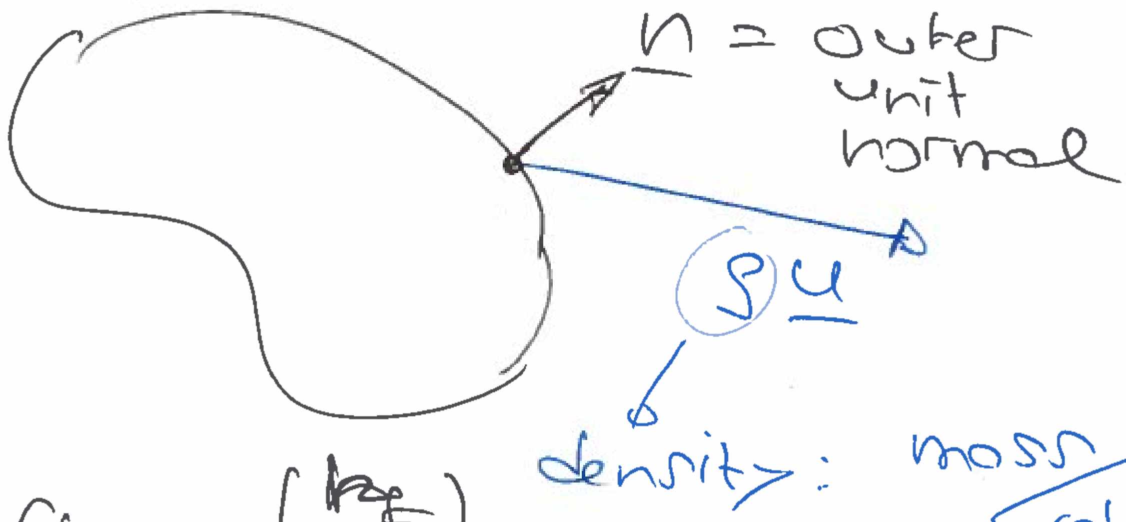
dilation & shearing

$$u_i(x_k+\delta x_k) = u_i(x_k) + \omega_{ij}\,\delta x_j + \varepsilon_{ij}\,\delta x_j$$

Eqn. of continuity

Physics: mass flux into a spatially fixed (control) volume = rate of change of mass in that volume

Integral form:



mass flux: $\left[\frac{kg}{sec}\right]$ density $\rho \left[\frac{kg}{m^3}\right]$ × velocity normal to the boundary $\left[\frac{m}{sec}\right]$ × area $[m^2]$

$$\underbrace{-\oint (\rho\,\underline{u})\cdot\underline{n}\,dA}_{\text{Div. th'm}} = \frac{d}{dt}\int \rho\,dV = \int \frac{\partial\rho}{\partial t}\,dV$$

$$-\int \mathrm{div}(\rho\,\underline{u})\,dV$$

$$\int \left[\frac{\partial\rho}{\partial t} + \mathrm{div}(\rho\,\underline{u})\right] dV = 0$$

but this has to hold for <u>any</u> control volume

$\Rightarrow$ integrand has to vanish!

$\Rightarrow$ differential form:

$$\boxed{\frac{\partial\rho}{\partial t} + \mathrm{div}(\rho\,\underline{u}) = 0}$$