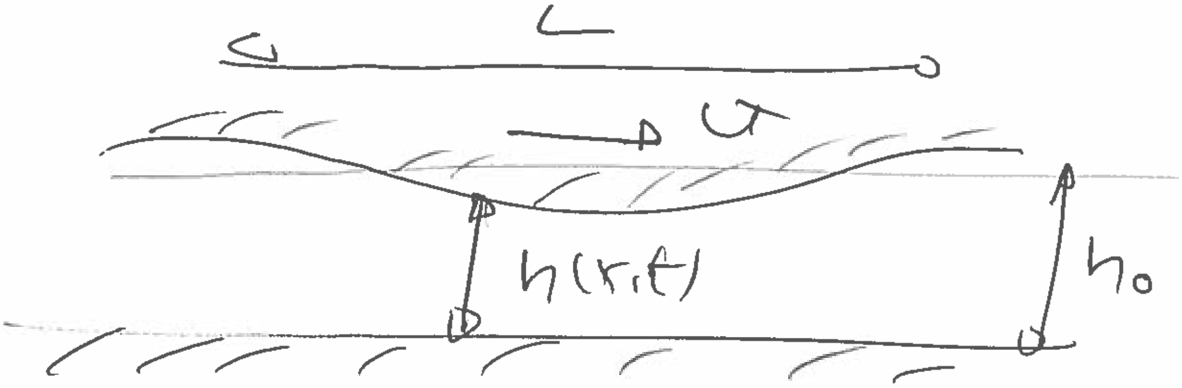


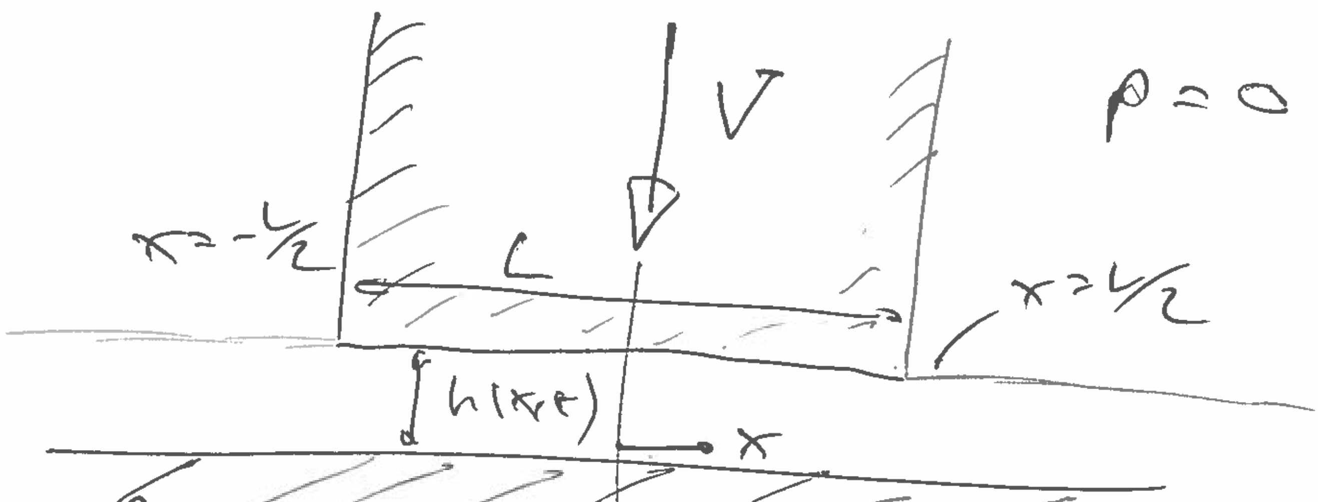
$h_0/L \ll 1$

$$u(x,y,t) = \underbrace{\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2 - y\,h(x,t)\right)} + \frac{U}{h(x,t)}\,y$$

$$\boxed{\frac{\partial}{\partial x}\left(\frac{h(x,t)}{\mu}\frac{\partial p}{\partial x}\right) = 12\frac{\partial h}{\partial t} + 6U\frac{\partial h}{\partial x}}$$

Reynolds' lubrication eqn
PDE for $p(x,t)$

Example: Squeeze film:



Assume: $p(x=-L/2) = p(x=L/2) = 0$

$$\frac{\partial h}{\partial t} = -V \; ; \quad h(x,t) = h_0 - Vt \; (2$$

$$\frac{\partial h}{\partial x} = 0$$

Into PDE:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) = 12\,\dot{h}$$

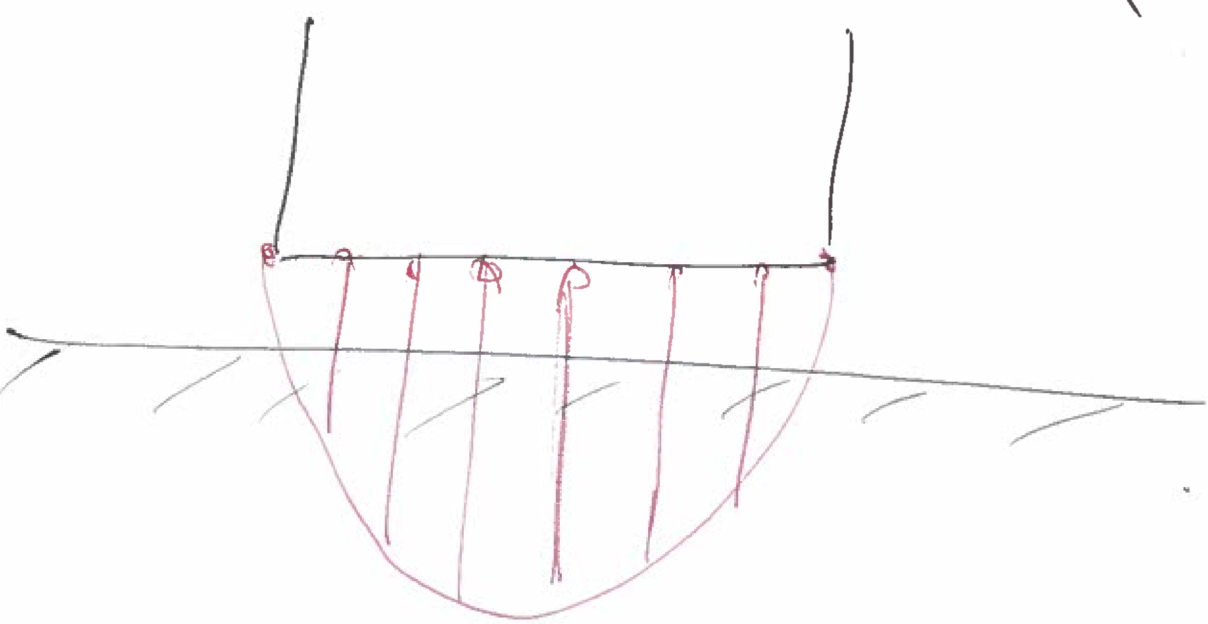
$$\frac{h^3}{\mu}\frac{\partial p}{\partial x} = 12\,\dot{h}\,x + \tilde{A}(t)$$

$$p(x,t) = \frac{12\,\dot{h}\,\mu}{h^3}\,\frac{x^2}{2} + A(t)\,x + B(t)$$

Apply BCs $\longrightarrow$ $A(t)$ $B(t)$:

$$p(x,t) = \frac{6\,\dot{h}\,\mu}{h^3}\left(x^2 - \left(\frac{L}{2}\right)^2\right).$$

- parabolic press. distribution



Temporal variation comes from $1/h^3(x,t)$

$$p(x,t) \sim \frac{1}{(h_0 - Vt)^3}$$

<u>Note:</u> As $h \to 0$ $\left(t \to \frac{h_0}{V}\right)$ pressure becomes ∞!

Alternatively: If a constant force is applied to the "stamp" the it would take ∞-long for $h \to 0$.

$\Longrightarrow$ "Lubrication".

THANK YOU

&

MERRY CHRISTMAS

!

... AND NOW ALL IN THE YEARS !

- Kinematics: ε_{ij} ω_{ij} interpretation.
- Parallel flow: deriv., apply, volume flux

$$q = \int u \, dy \quad \text{(per unit depth)}$$

- All eqns in cartesians: N. St., ε_{ij}, ω_{ij}, $\nabla^4 \psi = 0$

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

All given in other coord. systems.
- Dim. analysis
- NSt examples, BCs.
- Stokes $\nabla^4 \psi = 0$
  - Solve PDE
  - construct soln from known fen. soln.